THE TEACHING OF ARITHMETIC IN PRIMARY SCHOOLS

HANDBOOKS FOR UGANDA TEACHERS

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PREFACE

TO THE SECOND PRINTING

In April 1957 a group of teacher-trainers, District Education Officers and Supervisors assembled to consider the problem of arithmetic teaching in schools in Uganda. For some time those connected with the schools had been uneasy for it was considered that children were not getting a sound enough foundation in arithmetic. At the same time it was noted that considerable progress had been made in the schools in English and this was attributed by many to the great assistance given to the teachers by the detailed notes published together with the pupils' books. The fact that students were thoroughly trained in the use of these books during their training also improved considerably the teaching of the subject. It was felt, in consequence, that if something similar could be done for arithmetic, one had every reason for hoping that the results would be just as satisfying. The authors, therefore, who are all people with considerable experience in training colleges and in the field, have attempted to analyse the subject in a way that has not been done before for teachers in Uganda. Experience has shown that it is too much to hope for an analysis of existing textbooks to be made within the colleges, but the fact remains that it is essential for a teacher during his training to be led to understand the grading of work and the causes of the mistakes made by the children. There is considerable detail, but this is only because it is known that teachers require a great amount of guidance, and it is hoped that as a result of thorough study of this handbook teachers will come to have a greater interest in, as well as understanding of, this subject. They will be further helped by the pupils' books which will be published in due course.

Pupils' books are being prepared, but until the series is complete it is considered that with the amount of guidance given in this handbook teachers will be able to prepare sums correctly themselves.

It will be noted that no reference is made to Class 1 work because it requires a different approach and this is clearly explained in the many admirable books available on the subject of infant method. Only with the presence of many six-year-olds in Class 1 has it been possible to introduce this approach in Uganda schools in recent years, and as the children become younger it will become necessary to teach more realistically in Class 1, for it is most important that in this year children should have a thorough understanding of number.

It will be noted that the emphasis is very definitely on mechanical work. This is being given increasing importance in the United Kingdom and it has become clear that what is needed in Uganda is considerable practice by the pupils in the doing of mechanical sums. The difficulties in the way of producing suitable books in the many vernaculars and the tendency to spend too much time on simple problems before mechanical work is fully understood has prompted the writers to put more emphasis on mechanical work. It is encouraging to note that already, as a result of working on the lines of this book, children are showing a better grasp of arithmetic. They are completing many more sums than formerly, and they are getting more satisfaction out of the subject. In Classes 5 and 6 there will, of course, be problems, but it is most important for teachers to ensure that children have a sound grounding in mechanical rules and that when problems are introduced the language is as simple as possible and can be understood by the children. It is intended to give guidance on problems, and also mental arithmetic, in a further publication.

INTRODUCTION

(I) GENERAL RULES

This book has been written to give teachers a clear idea of the steps by which arithmetic should be taught. There are also set out at the very beginning the following basic principles which should help teachers in their work :

1. The importance of mechanical drill in each step of each type of sum must be realised. At present far too little time is spent on mechanical work in the first 4 classes and far too much time on problem work which, in fact, is often nothing more than copying a sum from the blackboard. Teachers will get better results in arithmetic if mechanical drill is stressed.
2. It is vitally important that children should get as many sums as possible correct. Teachers should, therefore, supplement the sums given in this book until such time as all children are getting the sums correct.
3. There should never be less than 6 sums to be done in each lesson and each step must be practised until it is done properly by the whole class.
4. A final exercise should always be done if the last step for a particular sum does not include all previous steps, e.g. the step of zero difficulties in the division of number will be followed by an exercise with the division of number in which some sums only will have zero difficulties.
5. By careful correction make sure that slower children are given every help to keep up with the main body of the class.
6. When putting exercises on the blackboard always put one more than the best child is thought capable of doing in the lesson so that there is no waste of time and children are required to put forth their maximum effort to complete all sums.
7. Always revise any process which is to be taught a stage further, e.g. when about to do addition of 3 digits give a revision of 2 digit addition.

8. Each teacher must be sure to complete his syllabus by the end of the year.
9. Headmasters, by careful supervision and periodical staff meetings, must ensure that the work of the syllabus is being properly covered.
10. At the end of term all class teachers should report on the stage they have reached in their syllabus so as to enable the head teacher to check on any failure to keep up with the syllabus and urge or help them to complete it by the end of the year.

(II) RULES FOR WRITTEN WORK

Untidy and ugly written work is chiefly due to a lack of uniformity in schools, and of knowledge by the teachers as to what is the accepted method of writing out sums and their working on the page; thus, teachers have often given no guidance so that children have not been helped to achieve a higher standard in general neatness.

If teachers observe the following rules and insist on the children doing the same, they will be pleased with the improvement in the childrens' work. Teachers should insist that they be carried out to the maximum ability of each child and no variations or indifferent work should be accepted by any teacher.

1. Children must not use a rough second book, nor odd scraps of paper, nor must they be allowed to scribble on covers and previous pages of the exercise book in use. All work that is not mental must be written and worked out neatly in the body of the sum.

2. No right-hand margin may be used for calculation because it encourages untidiness, mistakes through bad figures and a lazy mental attitude.

(Apart from the importance of neatness, the rule of all work within the sum has further significance in that it enables the teacher to mark exercises correctly—this is further explained in the section on Marking and Corrections.)

3. The abbreviation '*Ans.*' for Answer must be written opposite the answer wherever it is found in the particular type of

sum (see examples in these notes). The abbreviation *Ans.* should only be introduced in Class 3 as this is the stage when the sums are getting more complex.

4. In Class 2 half-size exercise books are more suitable for the children. The children should be shown how to fold the page down the middle ; they should *not* draw a line. On each side of this fold there is room for two sums to be neatly done. In Class 3 too, so small are the sums in the early part that a fold may well be required, though by this stage if teachers have taught their children well, neat work will have become established and a fold unnecessary. A full-size exercise book should be used in Class 3.

In Class 2 horizontal lines must not be drawn to separate sums ; an empty line must be left between sums. It is neater and less wasteful of time. Young children find line drawing difficult to do well and it takes far too long.

5. Sums of two quantities and more must be widely separated between quantities ; e.g. there must be 3 or 5 squares between yd. and ft., ft. and in. When the under-line working increases in later classes, it will be found necessary to increase to 5 squares. This will enable the under-line working of the sum to be done in adequate space.

Shillings and cents must be separated by a dot *on* the line.

6. The way the sums are done and set out in the examples of these notes must be followed completely to get the best results.

7. The signs + − × ÷ must always be inserted at the lower left (see examples and paragraph 11), and should not be omitted. In the case of addition and subtraction sums it will be on the left of the left quantity column ; in the case of multiplication, on the left of the multiplier, e.g.

	yd.	ft.	in.	
	6	2	9	
+		2	5	
				Ans.

yd.	ft.	in.	
6	2	9	
		×5	
			Ans.

The practice of inserting the signs and figure twice is unnecessary and must be discouraged—it should only be placed in its working position, e.g.

yd.	ft.	in.	
6	2	9 × 5	*Wrong*
		× 5	
			Ans.

The top × 5 is unnecessary, and must not be written.

The equal sign must be correctly used at all times, e.g.

How many socks in 2 pairs and an odd 1?

$2 \times 2 = 4 + 1 = 5$ is wrong.

It should read as :

$$2 \times 2 = 4$$
$$4 + 1 = 5$$

8. The abbreviations used in this book should be used at all times, and no other versions allowed. When figures and abbreviations are written horizontally, the abbreviation follows the figure, e.g. 3 yd. 2 ft. 5 in. : 2 lb. 6 oz., except in the case of £. Where shillings are mentioned without cents, and where cents are expressed as part of a shilling by means of a decimal point, the abbreviation sh. should be written before the figures, e.g. sh. 99, and sh. 99·90, but when the word cents is written the abbreviation sh. should follow the figure, e.g. 99 sh. 90 ct.

9. Squared paper must be used up to the end of Class 4. At that stage children should have achieved a high standard of neatness and be able to continue in the next classes equally well on narrow lined paper (not the wider lined writing paper).

It is important that teachers *insist* on good figures, one figure within one square. When marking, teachers should look closely for cases of bad figures and bring the attention of individual children to them.

10. Remainders in sums should be indicated as shown in the examples by the letter ' r '.

11. When sums are written on the blackboard, the children must be taught to write the sum down in the way it is to be worked and on no account must it be written in both ways.

(*a*) $104+29+6+129$ must be copied down *immediately* in the exercise book as:

$$\begin{array}{r} 104 \\ 29 \\ 6 \\ +129 \\ \hline \\ \hline \end{array}$$

(*b*) $385 \div 6$ as $6\overline{)385}$

(III) RULES FOR MARKING AND CORRECTIONS

1. *The correction* of all mistakes is essential. These should be done in the next period. Some of the *marking* should be done while the exercise is in progress, when the teacher should be going round examining the children's work, the rest of the marking being done in out-of-school hours.

2. The teacher must indicate the error as well as marking the sum wrong. It is *not enough* for the teacher merely to look at the answer, mark right or wrong, and pass on to the next sum. If the sum is wrong, the teacher must look for the error. As there are two main kinds of errors, there should be an indication of the kind of error it is. They are:

(*a*) the careless error which should be indicated by the teacher crossing through the error;

(*b*) the error of inability to understand the process or method of the sum when the teacher should place a bold T at the left-hand side of the error.

This is made clear in the following examples:

(*a*)
$$\begin{array}{rl} 39 & \\ \times 16 & \\ \hline 224 = & 6 \times 39 \quad \text{X} \\ 390 = & 10 \times 39 \\ \hline 614 = & 16 \times 39 \\ \hline \end{array}$$

(*b*)
$$\begin{array}{lrl} & 39 & \\ & \times 16 & \\ \hline & 234 = & 6 \times 39 \quad \text{X} \\ \mathbf{T} & 39 = & 10 \times 39 \\ \hline & 273 = & 16 \times 39 \\ \hline \end{array}$$

3. In the following lesson, those children with no errors will proceed with an exercise of similar character to the last exercise;

group (*a*) will correct those sums wrong and should have no difficulty in dealing with this careless error ; group (*b*) will be brought out to the blackboard and retaught. It is recommended that teachers, when they are marking should note down on paper the types of errors they come to, so that when they reteach group (*b*) they can lay the necessary stress on them.

4. It is urged upon teachers to regard this part of their work as of the greatest importance as it is unfortunately common to find classes with very many sums wrong and yet the teacher goes on through the syllabus. This is very unjust to the children and makes a complete failure of the teacher's work and that of succeeding teachers who are building on very bad foundations.

5. Each step, as indicated in the handbook must be thoroughly practised and drilled until all children are able to do it at good speed. (**Revision sums must not be given to children during these periods of practice in each step.**) Revision sums should be given at convenient places ; for example, when a teacher in Class 3 is teaching Measure (yd. ft. in.) addition he should complete the teaching of this before any revision sum is given to the children. When the class is quite able to do this type of sum, then the teacher could give a period of revision sums in the form of a test, if he so wishes. Too great stress cannot be placed on this paragraph as many teachers are now daily giving mixed sums and it is far too common to find a teacher, who has just taught a particular kind of sum, to be giving only one of these day after day mixed up with others learnt in previous lessons.

For example, the following type of daily exercise is often seen :

(i) $921+5067+98+1052$

(ii) 67×23

(iii) Daudi collected 20 eggs, but fell and broke a quarter of them. How many were whole?

(iv) 2 yd. 2 ft. 6 in. $\times 5$

The teacher had been teaching multiplication of yd. ft. in. Yet the children were doing but one each day for a number of periods and therefore having insufficient practice in them before going on to a new type of sum. The children should have been doing at least six sums daily in multiplication of yd. ft. and in., as will be made clear in this book.

SYLLABUS

on which this Handbook is based

The division of work into terms is given as a guide, and experience may show the advisability of making small changes. The ultimate point to be reached in each item is indicated.

CLASS 2

TERM 1. Addition of Number ; 2 digit numbers to a maximum total of 99.

Subtraction of Number ; 2 digit numbers, maximum top number of 99.

Simple 2 item Bills (Mental).

Addition of Money—shillings and cents going up to a total of 19 sh. 90 ct. in 3 items, giving cents in multiples of 10 only.

TERM 2. Short Multiplication of Number ; 2 digits only within the known Tables.

Short Division of Number ; 2 digits only within the known Tables.

Simple Bills involving subtraction (Mental).

Subtraction of Money, with a maximum of 29 sh. 90 ct. in the top line.

TERM 3. Short Multiplication of Money within the known Tables.

Linear Measurement—Practical work.

Bills (Written)—2 items only.

Addition of Number ; 3 digit numbers to a maximum answer of 999, going up to 4 items.

CLASS 3

TERM 1. Subtraction of Number ; subtraction of 3 digits from 3 digits.

Short Multiplication of Number, using all Tables going up to 3 digits in the top line and 999 in the answer

Short Division of Number, using all Tables and going up to 3 digits in the dividend.

Addition of Money, introducing 5 cents, then 1 cent, going up to 4 items. Answer not to be more than sh. 99·99.

Subtraction of Money, introducing 5 cents and 1 cent.

TERM 2. Short Multiplication of Money, using all Tables; answer not to exceed sh. 99·99.

Short Division of Money, using all Tables, the dividend not to exceed sh. 99·99.

Linear Measurement. Simple 4 rules in yd. ft. and then in yd. ft. in. having yd. in answer only.

Fractions. Practical work introducing $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$ and $\frac{1}{6}$. Simple addition and subtraction.

TERM 3. Long Multiplication of Number, 2 digits only in both lines.

Capacity. 4 rules in debes, gallons and pints.

Weight. Addition and Subtraction lb. oz.

Simple bills that can be done mentally.

Telling the time in the vernacular.

Fractions—introduction of $\frac{1}{8}$ and simple fractional parts.

CLASS 4

TERM 1. Addition of Number, going up to 5 items, with a maximum of 5 digits in the answer.

Subtraction of Number, maximum of 5 digits on top line.

Short Multiplication, maximum of 4 digits on top line.

Long Multiplication by 2 digits, going up to 3 digits in the top line.

Short Division, going up to 5 digits in dividend.

Long Division of Number, 4 digits in dividend.

Addition of Money, up to 5 items, answer not to exceed sh. 99,999.

Introduction to Profit and Loss.

TERM 2. Fractions. Introduction of $\frac{1}{12}$, $\frac{1}{9}$, $\frac{1}{6}$ and $\frac{1}{10}$. Addition and subtraction; Improper and Proper fractions: addition of 3 items.

Short Multiplication of Money, top line not to exceed Sh. 999 in sh. column.

Long Multiplication of Money, 2 digit multipliers and top line not to exceed 99 in sh. column.

Short Division of Money, dividend not to exceed sh. 999·99.

Long Division of Money, with 2 digit division and dividend not to exceed sh. 999·99.

TERM 3. Linear Measurement. Addition and Subtraction introducing chains and miles.

Linear Measurement: Short and Long Multiplication of yd. ft. in.

Short and Long Division of yd. ft. in.

Capacity. Introduce Quarts, in the 4 rules, with debes, gallons and pints.

Weight—Completion of 4 rules in lb. oz.

Perimeter.

CLASS 5

TERM 1. Money. 4 rules in £. sh.

Profit and Loss.

Linear Measurement.

Short and Long Multiplication with miles and chains.

Short and Long Division with miles and chains.

Dividing one measure into another.

Capacity—Long Multiplication and Long Division.

TERM 2. Weight. 4 rules, introducing ton, cwt. with lb. oz. also long multiplication and long division. Division of weight by weight.

Fractions. Multiplication and Division. Addition and subtraction of more than 2 fractions.

TERM 3. Introduction to Decimals. Addition and Subtraction.
Decimals. Multiplication and Division.
Unitary Method.
Work Sums.
Area.
Time.

CLASS 6

TERM 1. Money. £ and sh. increasing in difficulty.
Profit and Loss.
Percentage, changing fractions into percentage figures and percentages to fractions. Common percentages and their equivalent fractions; application to problems.
Percentage Profit and Loss.

TERM 2. Discount.
Interest.
Decimals—extension of Class 5 work to 4 places of decimals: decimal equivalents of vulgar fractions.
Proportion.
Work Sums—Inverse Proportion.

TERM 3. Revision.

In Class 2 multiplication tables must be learnt and daily drilled from the position reached in Class 1. The order of doing should be 2, 4, 10, 3, 6, 5, 11, 7, 8, 9, 12.

NUMBER

ADDITION

It must be understood that some of the earliest steps as given below will be found in the teaching of Class 1, *not Class* 2. *For convenience, they have been listed under Class* 2. *The abbreviations Th (Thousands, H (Hundreds), T (Tens) and U (Units) have been used throughout.*

Aim

(a) Addition of 2 digit figures to give totals up to a maximum of 99. (b) The carrying figure to be established—it must be inserted neatly until such time as the children can be encouraged to do without it.

The teacher must ensure that he moves forward in single steps of progression—only one new process must be taught, like this:

Step 1. The addition of U to U.

$$\begin{array}{r} 6 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +7 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +4 \\ \hline \end{array} \quad \text{etc.}$$

Step 2. In this step the principle of bundles of ten is introduced. Spent matchsticks or small sticks should be used. The children should be shown how to count out the required sticks, gather them together (add) and then separate the sticks into a bundle of ten, and ' so many ' sticks left over.

$$\begin{array}{r} 5 \\ +6 \\ \hline 11 \end{array} \quad \begin{array}{r} 8 \\ +9 \\ \hline 17 \end{array} \quad \begin{array}{r} 7 \\ +6 \\ \hline 13 \end{array} \quad \begin{array}{r} 8 \\ +4 \\ \hline 12 \end{array} \quad \text{etc.}$$

Step 3. The addition of U to TU with *no* carrying.

$$\begin{array}{rrrrrl} 15 & 14 & 12 & 11 & 16 & \\ +\ 3 & +\ 4 & +\ 7 & +\ 5 & +\ 3 & \text{etc.} \\ \hline \end{array}$$

Step 4. The addition of TU to TU with *no* carrying.

$$\begin{array}{rrrrrl} 15 & 14 & 12 & 21 & 26 & \\ +13 & +12 & +17 & +15 & +23 & \text{etc.} \\ \hline \end{array}$$

Step 5. The addition of TU to TU *with* carrying.

$$\begin{array}{rrrrrl} 15 & 14 & 15 & 26 & 28 & \\ +16 & +17 & +19 & +18 & +25 & \text{etc.} \\ \hline \end{array}$$

Step 6. When this has been grasped, 3 items should be added, and then 4 as :

$$\begin{array}{rrrl} 13 & 21 & 19 & \\ 19 & 16 & 5 & \\ +67 & +28 & +48 & \text{etc.} \\ \hline \end{array}$$

Step 7. The method of horizontal tots should be started here.

SUBTRACTION

Aim

The subtraction of two digit numbers, the maximum top number being 99.

Steps are as follows, starting with revision from Class 1.

Step 1. Subtraction of U from U.

$$\begin{array}{rrrl} 6 & 9 & 8 & \\ -3 & -4 & -3 & \text{etc.} \\ \hline \end{array}$$

Step 2. Subtraction of U from T only.

At this stage, practical demonstration is of very great importance. On a table or blackboard on a table in the middle of the class so that all the children can see, the teacher will write the sum

$$\begin{array}{r} 10 \\ -\ 3 \\ \hline \end{array}$$

On the figure 1 he will place a *bundle* of 10 sticks. The child, on his experience of Step 1, will be faced with the problem of $0-3$. He will get the answer that he must take the ten from the ten bundle, leaving nothing in its place. It is extremely important that this principle of taking or carrying is *understood* and is not merely a process that has to be done; this example should be repeated until it is grasped.

$$\begin{array}{r} 10 \\ -\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 10 \\ -\ 5 \\ \hline \end{array} \qquad \begin{array}{r} 10 \\ -\ 7 \\ \hline \end{array}$$

Step 3. Subtraction of U from TU, then TU from TU both with *no* carrying.

$$(a)\quad \begin{array}{r} 14 \\ -\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 17 \\ -\ 5 \\ \hline \end{array} \qquad (b)\quad \begin{array}{r} 19 \\ -14 \\ \hline \end{array} \qquad \begin{array}{r} 29 \\ -14 \\ \hline \end{array} \qquad \text{etc.}$$

Step 4. Subtraction of U from TU. The carrying figure is introduced a step further. This step requires care, and more time in teaching than the above.

The decomposition method, by which the 'borrowed' ten will be taken from the top line, will be the method to be adopted as this is the easiest method of practical demonstration. At first the decomposed digit will be neatly crossed through and the carried tens figure inserted neatly to the left of the unit figure.

$$\begin{array}{r} 1 \\ \cancel{1}4 \\ -\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ \cancel{1}7 \\ -\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ \cancel{1}5 \\ -\ 7 \\ \hline \end{array} \qquad \text{etc.}$$

Step 5. As Step 4, but increasing the T in the top line. The new decomposed figure will be inserted to the left of the cancelled ten.

$$\begin{array}{r} 1\ 1 \\ \cancel{2}3 \\ -\ \ 9 \\ \hline \end{array} \qquad \begin{array}{r} 1\ 1 \\ \cancel{2}4 \\ -\ \ 6 \\ \hline \end{array} \qquad \begin{array}{r} 2\ 1 \\ \cancel{3}7 \\ -\ \ 9 \\ \hline \end{array} \qquad \begin{array}{r} 3\ 1 \\ \cancel{4}3 \\ -\ \ 7 \\ \hline \end{array}$$

Step 6. Subtraction of one T and U from TU with carrying.

$$\begin{array}{r} 33 \\ -18 \\ \hline \end{array} \qquad \begin{array}{r} 45 \\ -16 \\ \hline \end{array} \qquad \text{etc.}$$

Step 7. Subtraction of TU from TU with carrying.

$$\begin{array}{rrr} 37 & 53 & 63 \\ -29 & -28 & -45 \\ \hline \end{array}$$

It is repeated that the decomposed figure will be crossed out and the lower figure inserted as shown in the previous examples. This will cease as each child is deemed capable of dealing with it mentally. The carried ten will be inserted again on the left of the unit figure and will be regarded with the unit as one complete number, as follows:

$$\begin{array}{rr} & 2\,1 \\ & \cancel{3}3 \\ - & 18 \\ \hline & 15 \\ \hline \end{array}$$

Thus the child takes 8 from 13.

Step 8. Give a few final exercises containing sums that are mixed, some with carrying and some without.

BILLS

The addition of money will have been started by means of the 'class shop' in Class 1. In this way the idea of bills will not be new to the children. In this shop a number of articles should be placed together on view before the class, such as a hoe, basket, pot, gourd, mat, pail, etc., with a price ticket on each. Each article will be priced as 10 ct. 20 ct. 30 ct. etc., or in shillings up to 10. (*Not* in sh. and ct.) Only 1 of each article is bought and only 2 articles so that there is no multiplication, but only addition of 2 items of money. The child buys the 2 articles and is required to say how much money he must pay.

MONEY

Money should follow number because it is an easy introduction to those sums requiring change of unit, i.e. weight, capacity, etc.

Three squares should separate the figures between shillings and cents. A dot *on* the line in the centre square should be used to separate shillings from cents.

Revise practical work on 10 and 50 cent pieces, done in Class 1, by means of a shop.

ADDITION

Aim

The addition of shillings and cents in 3 items, up to 19 shillings 90 cents, giving cents in multiples of 10 only.

Step 1. Adding cents (10 cent pieces only) to a total of not more than 90 cents :

	sh.	ct.
		50
+		30

	sh.	ct.
		20
+		40

	sh.	ct.
		70
+		10

	sh.	ct.
		30
+		60

	sh.	ct.
		10
+		80

	sh.	ct.
		20
+		20

etc.

Step 2. Adding cents (10 cent pieces only) to a total of more than one shilling :

	sh.	ct.
		50
+		50
	1	00
	1	100=1.00

	sh.	ct.
		60
+		50
	1	10
	1	110=1.10

	sh.	ct.
		90
+		70
	1	60
	1	160=1.60

	sh.	ct.
		20
+		80
	1	00
	1	100=1.00

Step 3. Addition of cents and shillings and cents, the cents not totalling more than 90 ct. ; with no carrying from cents to shillings.

	sh.	ct.
	1	30
+		40

	sh.	ct.
	1	60
+		20

	sh.	ct.
	1	00
+	1	70

	sh.	ct.
	2	70
+	1	10

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
1	40	5	30	10	70	14	10
+ 3	40	+ 7	30	+ 5	20	+ 5	80

Step 4. Addition of shillings and cents with carrying from ct. to sh.

sh.	ct.	sh.	ct.	sh.	ct.
1	50	4	70	6	80
+ 1	50	+ 5	50	+ 9	70
3	00	10	20	16	50
1	100 = 1.00	1	120 = 1.20	1	150 = 1.50
2		9		15	
3		10		16	

Step 5. As Step 4 but increase to 3 items.

NUMBER

MULTIPLICATION

Continue with multiplication tables. It is emphasised that table drill must be thorough and plentiful, and every opportunity must be taken to use spare moments in the day in class recitation and individual recitation.

Aim

Short multiplication of number ; 2 digits only within the known tables.

Step 1. The multiplication of U.

$$\begin{array}{rrrl} 6 & 9 & 8 & \\ \times 2 & \times 4 & \times 6 & \text{etc.} \\ \hline \end{array}$$

Step 2. Multiplication of TU with *no* carrying.

$$\begin{array}{rrrl} 13 & 13 & 24 & \\ \times 2 & \times 3 & \times 2 & \text{etc.} \\ \hline \end{array}$$

Emphasise the position of the multiplying figure.

Step 3. Multiplication of TU *with* carrying figure. The carrying figure will be inserted under the line as shown. Later this practice can cease as each child shows himself capable of doing without ; it is better to delay this, than allow it too soon. At this stage, the maximum answer must be 99.

$$\begin{array}{rrrl} 13 & 24 & 26 & \\ \times 4 & \times 4 & \times 3 & \text{etc.} \\ \hline 52 & 96 & 78 & \\ \hline 1 & 1 & 1 & \end{array}$$

DIVISION

Aim

Short division of two digits within the known tables.

Step 1. The division of TU with *no* carrying figure.

$2\overline{)48}$ $3\overline{)69}$ $4\overline{)48}$ etc.

Step 2. Division of TU with carrying figure. The carrying figure will be inserted neatly above and between the 2 digits, and dropped as soon as it is regarded as within the ability of the children.

$2\overline{)5^{1}8}$ $3\overline{)8^{2}1}$ $4\overline{)7^{3}6}$ etc.

Step 3. Division in which the divisor is greater than the T figure.

Emphasise that the answer figure must be in its correct position *under* the unit figure.

$4\overline{)36}$ $5\overline{)35}$ $6\overline{)42}$ etc.

The zero must not be brought into any examples of these exercises.

BILLS

The work will carry on as in Term 1; but children will now be introduced to simple mental subtraction through the giving of change out of sh. 1, sh. 10 and sh. 20.

MONEY

SUBTRACTION

Aim

Subtraction of money with a maximum of 29 sh. 90 ct. in the top line.

Here is the first really substantial introduction into the changing of the unit, i.e. borrowing from a different quantity. The 0 must be written in the cents column where necessary; a blank must not be left. The top line of the subtraction should never exceed sh. 29·90.

Proceed gradually according to the steps as follows:

Step 1. Subtraction of cents (10 cents only) without carrying:

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
	30		50		90		80
−	20	−	10	−	60	−	30

sh.	ct.	sh.	ct.	
	70		50	etc.
−	40	−	30	

Make up more examples for class work.

Step 2. Subtraction of cents (10 cent multiples) from shillings and cents by decomposition: it should be made clear to the children that when transferring one shilling into the cents column, it becomes 100 cents and that by placing a one in front of the cents numbers, this number becomes one hundred and so many cents, i.e. 50 ct. becomes 150 ct. after carrying the sh. across to the ct. column.

	sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
(*a*)	0	1	0	1	0	1	0	1
	~~1~~	00	~~1~~	00	~~1~~	00	~~1~~	00
	−	30	−	90	−	50	−	60
		70		10		50		40

	sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
(*b*)	0	1	0	1	1	1	1	1
	~~1~~	70	~~1~~	50	2	00	2	60
	−	80	−	80	−	40	−	80
		90		70	1	60	1	80

Make up more examples, based on these examples for class work, gradually increasing the number in the shillings column. Do the other five in Section (*a*) before proceeding to Section (*b*).

Step 3. Subtraction of shillings and cents (10 cent multiples) from shillings and cents without carrying.

	sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
(*a*)	1	20	1	80	2	00	2	50
	− 1	10	− 1	40	− 1	00	− 1	30

sh.	ct.		sh.	ct.
4	90		8	70
– 2	20		– 5	40

Make up more examples on these lines.

(*b*) Subtraction of sh. ct. from sh. ct. with carrying *within* the sh. column, but not from sh. to ct. It is essential at this stage that the class be able to take away in Number without inserting the carrying figure; otherwise there will be too much crossing through and insertions in the next step when carrying is introduced a step further.

sh.	ct.		sh.	ct.
13	60		27	40
– 7	30		– 9	10
6	30		18	30

Step 4. Subtraction of shillings and cents from shillings and cents with carrying from the shillings column to the ct. and no carrying within the sh. column at first.

	sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
(*a*)	1	1	2	1	4	1	7	1
	~~2~~	20	~~3~~	70	~~5~~	20	~~8~~	10
	–1	50	–1	90	–3	60	–2	30
		70	1	80	1	60	5	80

	sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
(*b*)	9	1	2	1	7	1	0	1
	1~~0~~	00	1~~3~~	70	1~~8~~	50	2~~1~~	00
	– 6	50	–12	80	– 6	80	–14	30
	3	50		90	11	70	6	70

Make 20 more examples for class work, 10 of (*a*) and 10 of (*b*). The sums should be done as shown at first, but later when the process is mastered, the figure insertion should be dropped.

MONEY

SHORT MULTIPLICATION

Aim

Short multiplication of money within the known tables, answers not exceeding sh. 99 and ct. 90, and ten cent pieces only to be used.

It is essential that the tables are well known and that the children have fully grasped the method of addition and subtraction before starting on multiplication.

The following stages to be followed :

Step 1. Multiplication of cents (10 cent multiples) by known tables:

Step (*a*) has no carrying figure—one period is sufficient.

(*a*)

sh.	ct.
	30
	×2

sh.	ct.
	20
	×4

(*b*)

sh.	ct.
	40
	×3
1	20

120 = 1.20

sh.	ct.
	60
	×6
3	60

360 = 3.60

sh.	ct.
	80
	×7
5	60

560 = 5.60

Make 20 more examples for class work.

Step 2. Multiplication of sh. and ct. (10 cents) by known tables; up to a maximum of sh. 99 and ct. 90.

sh.	ct.
1	00
	×2
2	00

sh.	ct.
1	20
	×8
9	60
1	160 = 1.60
8	
9	

sh.	ct.
2	20
	×9
19	80
1	180 = 1.80
18	
19	

sh.	ct.
4	70
	×4
18	80
2	280 = 2.80
16	
18	

sh.	ct.	
6	60	
	×5	
33	00	etc.
3	300=3.00	
30		
33		

Make 20 more examples, similar to the ones above, for class work.

LINEAR MEASURE

Aim

Children to be given practical experience of measuring and estimating length.

Step 1. Begin with a yard-stick or rope of 1 yd. length, or reed, plaited banana fibres etc. Tell the class ' This is used to measure the length or width or height of anything which is fairly big. It is called a *yard*. Anything called a *yard* must be of exactly this length '. Children should have one yard-measure between two or three of them : and should then go in groups to measure as many objects as possible, writing down the object and its measurement on paper, thus :

> The end wall of the school is (exactly) (nearly) (just over) *X* yards long.

(Teachers may use this information for handwriting exercises.)

Step 2. After one or two such periods, the teacher should mark off into 3 feet all the children's (and his own) yard-measure. Tell the class ' When we measured (something just over or nearly *X* yards) we found that our yard did not measure the .... exactly. If we want to know how much remains after we have used a whole yard as often as possible, we use a small stick, or rope, whose length is called a *foot*. (Hold up a home-made 1 foot ruler.) Every *foot* is exactly this length. If I measure my yard with my foot, I see that I have to use my foot three times ' (Here call two or three boys to measure the teacher's *yard* with his *foot-rule*) ' So every yard has three feet '. Here give each group

its marked-off yard-measure, and let them *see* that 1 yard = 3 feet. Then the class should measure again the objects measured previously with the yard and record their answers as:

The side of the compound nearest the road is

(*a*) X yards and nearly Y feet

or (*b*) X yards and just over Y feet

or (*c*) exactly X yards and Y feet

or (*d*) exactly X yards.

N.B. *Do not* give the children a marked foot-rule of their own at Stage 2.

Step 3. Teacher holds up a foot, revises the names—'3 feet = 1 yard'. Tell the class: 'We said that we used a yard to measure fairly big things. Sometimes we have to measure things which are not as long as one yard, or only a little more than one yard.' (Call out a boy and measure the yard against him.) 'For this sort of measuring we use a foot by itself.' Give the class a stick each, measuring one foot. Then let them measure one another and record their results as follows:

William is exactly 4 feet tall.

Musa is nearly 4 feet tall.

Mary is just over 3 feet tall.

Groups of two or three should then measure each desk, the teacher's table, the cupboard etc., and record their results in the same way.

After two or three periods using the foot only, return to measurement using the yard measure and the foot measure (by different groups) for one period.

Step 4. Distribute home-made rules. An ordinary foot-rule should *never* be used because:

(*a*) There are too many marks on it.

(*b*) The *ends* of such a ruler will lead to mistakes and confusion.

Therefore the class teacher should make cardboard rulers 12 inches *exactly* in length (and one inch in width) marked off accurately in inches. Revise name of foot-rule. Then say:

'When we measured with our yard, we found that we could not measure the ...., or the .... exactly: so we used yards and feet: when we measured with our foot, we found that we could not measure ....'s height or the length of ....'s desk exactly. Now look at the foot-rule which you have on your desk.' On your ruler you can see a number of lines. These lines have a figure written by the side. What figures can you see? 'Well the distance from the line called "1" to the line called "2" is *one inch*. We use an inch to measure how much remains after we have used a whole foot as often as possible. The distance from the beginning of the ruler to figure 1 is 1 inch. From figure 1 to figure 2 is one inch: What is the distance from the beginning of the ruler to figure 2?' (Here build up the reading of the ruler as carefully as the intelligence of the class requires.) 'In the whole foot there are 12 inches: so one foot equals 12 inches.'

Now let the children measure one another and the objects previously measured in feet only, and record their results as follows:

William is 4 feet tall.

Musa is 3 feet and 10 inches tall, etc.

Step 5. Further periods of practical measurement in feet and inches. Then hold up an object and say: 'Sometimes a thing is not even one foot long. How do you think we can measure such an object?' The children by now will be able to answer, and this can be followed by the measurement of small objects in inches only.

Note carefully:

1. Do not allow abbreviations (yd. ft. in.) *nor use them* on the blackboard in this class.

2. A useful piece of permanent apparatus is a piece of board about 5 ft. in height, fastened to the wall or a door post so that the class may measure one another throughout the year, thus continually revising their knowledge.

3. The *table* 1 yd. = 3 ft., 1 ft. = 12 in. should be revised periodically in Mental Arithmetic or Tables time.

BILLS

Aim

To introduce children to the written form of bills, which will, of course, be more advanced.

On the wall there should be placed pictures of suitably priced articles in multiples of 10 ct. and sh. and ct. (multiples again of 10)—low numbers of shillings, e.g. 60 ct., 1.20, 2.60. Children will now be required to buy as many as 3 of each article and may buy 2 articles (no more), e.g. 3 pencils and 3 pens.

The setting down of the bills must be carefully explained. Make squares on the blackboard to represent exactly a page of the exercise book, and give directions on the exact position of each heading, i.e. in which square each heading starts. Show how the number or wording in each column starts in the same square as the second letter of the column word so that the figures and words of each column are in a neat vertical shape; e.g. ensure that units go under units, tens under tens:

NUMBER	ARTICLE	COST OF 1	COST OF ALL
1	Bucket	4.00	4.00
3	Hoe	5.20	15.60
		Total cost	19·60

If this is clearly explained, you will be sure to get neat work from the class.

When the class is given the written work to do go round supervising and guiding the children. Sums should be separated by an empty line—there is no need to repeat the headings except when a new page is started.

NUMBER

ADDITION

Aim

The ultimate point to be reached will be the addition of 3 digit numbers to give a maximum answer of 999. (The steps will be as follows, after revision.)

Step 1. This introduces a H in one item with carrying from U to T only (one period).

$$\begin{array}{r} 143 \\ +\ 49 \\ \hline \end{array} \quad \begin{array}{r} 128 \\ +\ 67 \\ \hline \end{array} \quad \begin{array}{r} 136 \\ +\ 57 \\ \hline \end{array} \quad \begin{array}{r} 258 \\ +\ 29 \\ \hline \end{array} \quad \begin{array}{r} 367 \\ +\ 19 \\ \hline \end{array}$$

Step 2. H in one item with carrying from T to H.

$$\begin{array}{r} 143 \\ +\ 62 \\ \hline \end{array} \quad \begin{array}{r} 152 \\ +\ 56 \\ \hline \end{array} \quad \begin{array}{r} 273 \\ +\ 66 \\ \hline \end{array}$$

Step 3. Carrying from both U and T.

$$\begin{array}{r} 177 \\ +\ 66 \\ \hline \end{array} \quad \begin{array}{r} 83 \\ +149 \\ \hline \end{array}$$

Step 4. Addition of HTU to HTU with carrying from T to H.

$$\begin{array}{r} 143 \\ +162 \\ \hline \end{array} \quad \begin{array}{r} 152 \\ +156 \\ \hline \end{array} \quad \begin{array}{r} 173 \\ +166 \\ \hline \end{array}$$

Note that here the carrying figure at first is restricted to the tens to hundreds column and that the carrying figure from both columns is left to step 5, so as not to give the children too much in the way of more difficult number manipulation while they deal with the new step.

Step 5. As step 4 with carrying from both U and T.

$$\begin{array}{r} 197 \\ +166 \\ \hline \end{array} \quad \begin{array}{r} 243 \\ +178 \\ \hline \end{array} \quad \begin{array}{r} 398 \\ +237 \\ \hline \end{array}$$

Step 6. The stages would be the addition of 3 and then 4 items as:

$$\begin{array}{r} 123 \\ 29 \\ +\ 86 \\ \hline \end{array} \quad \begin{array}{r} 156 \\ 72 \\ 429 \\ +184 \\ \hline \end{array}$$

At this stage, horizontal tots should be exercised up to a maximum of 3 items.

CLASS 3: TERM I

NUMBER

SUBTRACTION

Aim

The ultimate point will be the subtraction of 3 digits from 3 digits.

Step 1. The subtraction of TU from HTU with carrying from H to T.

$$\begin{array}{r} 136 \\ -\ 43 \\ \hline \end{array} \quad \begin{array}{r} 186 \\ -\ 92 \\ \hline \end{array} \quad \begin{array}{r} 237 \\ -\ 64 \\ \hline \end{array} \quad \begin{array}{r} 544 \\ -\ 74 \\ \hline \end{array} \quad \text{etc.}$$

Step 2. Subtraction of TU from HTU with carrying from H and from T.

$$\begin{array}{r} 136 \\ -\ 47 \\ \hline \end{array} \quad \begin{array}{r} 186 \\ -\ 98 \\ \hline \end{array} \quad \begin{array}{r} 237 \\ -\ 69 \\ \hline \end{array} \quad \begin{array}{r} 544 \\ -\ 77 \\ \hline \end{array} \quad \text{etc.}$$

Step 3. Subtraction of HTU from HTU with carrying in either or both columns.

$$\begin{array}{r} 560 \\ -237 \\ \hline \end{array} \quad \begin{array}{r} 634 \\ -197 \\ \hline \end{array} \quad \begin{array}{r} 225 \\ -170 \\ \hline \end{array} \quad \text{etc.}$$

Step 4. Introduce zero difficulties. Here the zero difficulty is one that is contained in the tens column of the top line. It is advisable at first to carry and insert the new figure in the top line even though dropped in previous work. It will help understanding. Put the first sum on the blackboard, and do it with the class. It illustrates the difficulty of needing to borrow a T from a column with no Tens. Ask the class how many Tens are in the number and elicit 20 (this *must* be understood by all). Elicit again

that if a T is borrowed there are 19 Tens left. So the blackboard reads:

$$\begin{array}{r} 19 \\ 2\cancel{0}6 \\ -\ 47 \\ \hline 159 \end{array}$$

$$\begin{array}{r} 206 \\ -\ 47 \\ \hline \end{array} \qquad \begin{array}{r} 207 \\ -\ 39 \\ \hline \end{array} \qquad \begin{array}{r} 407 \\ -198 \\ \hline \end{array} \qquad \text{etc.}$$

Step 5. Here the zero is contained in T and U of the top line.

$$\begin{array}{r} 9 \\ \cancel{1}\cancel{0}0 \\ -\ 37 \\ \hline \end{array} \qquad \begin{array}{r} 59 \\ \cancel{6}\cancel{0}0 \\ -\ 43 \\ \hline \end{array} \qquad \begin{array}{r} 49 \\ \cancel{5}\cancel{0}0 \\ -169 \\ \hline \end{array}$$

MULTIPLICATION

Aim

In this class all multiplication tables will be learnt thoroughly and short multiplication with all tables well practised in sums of up to 3 digits in the top line.

At this point it will be necessary to explain the thousand figure.

DIVISION

Aim

Short division of 3 digits using all tables in stages.

Step 1. Revise division of 2 figures *with* carrying figure (one period only is needed).

Step 2. The carrying from H to T. There is no carrying from T to U so as to concentrate attention on the new step.

4)564 7)847 6)846

Step 3. Carrying from both H and T.

(*a*) 6)732 4)652 5)735

(*b*) Division of HTU in which divisor is greater than H as:

6)528 7)525

Step 4. Introduce remainders beginning with 2 digit dividends:

4)58	3)89	5)88	etc.
14 r 2	29 r 2	17 r 3	

Step 5. As for Step 4, increase to 3 digits.

Step 6. Now, and not before, introduce zero difficulties in this order:

(*a*) a zero in unit figure of dividend and answer.

8)160	8)320	7)840
20	40	120

(*b*) a zero in unit figure of the answer with a remainder.

4)361	6)427	7)913	
90 r 1	70 r 1	130 r 3	etc.

(*c*) Before the next part, give an exercise on division sums with the above zero difficulties in some of the sums.

Step 7. (*a*) A zero in the T of the answer. In this sum children often forget to carry the T of the dividend.

4)436	7)735	8)856	
109	105	107	etc.

(*b*)

4)800	3)900
200	300

(*c*) A zero in T and U of the answer with remainder. Again give a few exercises on the Division of Number with some sums containing zero difficulties.

8)803	6)605
100 r 3	100 r 5

MONEY

ADDITION

Aim

The introduction of 5 cents and 1 cent. Children should dispense with the working under the line. Examples may

include up to 4 items but the answer is not to be more than sh. 99.99.

Step 1. Addition of 2 items using 5 cents with *no* carrying.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
6	25	4	15	7	50	11	25
+ 3	65	+ 6	05	+ 8	45	+15	60

sh.	ct.	sh.	ct.	
18	75	39	55	etc.
+24	05	+42	20	

Step 2. Addition using 5 cents *with* carrying.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
6	25	8	75	12	35	23	40
+ 3	75	+ 9	75	+17	65	+35	95

sh.	ct.	sh.	ct.	
49	25	87	35	etc.
+18	80	+ 9	75	

Step 3. Addition using 1 cent, with *no* carrying.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
5	26	9	54	19	75	26	67
+ 4	02	+11	31	+14	19	+57	24

sh.	ct.	sh.	ct.	
87	33	53	29	etc.
+ 8	48	+27	07	

Step 4. Addition using 1 cent *with* carrying.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
5	29	12	57	29	99	51	62
+ 7	78	+26	75	+47	47	+14	54

sh.	ct.	sh.	ct.	
76	77	4	83	etc.
+ 9	81	+57	39	

Step 5. (*a*) Now increase to 3 items and give exercises on addition in 3 items ; when well done, proceed to 4 items.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
16	50	22	37	36	24	78	19
42	35	42	16	17	02	2	75
+17	45	+13	46	+20	30	+10	67

sh.	ct.	sh.	ct.
37	91	7	07
29	87	15	21
+31	88	+26	74

(*b*) Exercises should include the carrying of more than 1 shilling.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
16	50	19	61	7	25	23	73
42	35	18	93	12	47	20	21
17	45	28	17	40	39	34	45
+ 6	95	+23	26	+ 9	20	+13	13

sh.	ct.	sh.	ct.	sh.	ct.
27	16	52	27	2	35
42	94	14	06	3	73
	25	2	14	4	64
+17	09	+ 8	75	+ 3	87

SUBTRACTION

Aim

The introduction of 5 cents and 1 cent. Children should dispense with crossing out and putting in the new figure as soon as they are able.

Step 1. Subtraction of cents (using 5 cents) from cents first (*a*) without carrying and then (*b*) with carrying.

	sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
(*a*)		75		55		95		35
	–	30	–	25	–	90	–	15

	sh.	ct.	sh.	ct.	sh.	ct.
(*b*)		70		50		40
	–	55	–	05	–	35

Make more examples based on these.

Step 2. Subtraction of cents (using 5 cents) from shillings and cents, carrying from the shillings column.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
1	00	1	25	1	65	2	15
–	75	–	65	–	70	–	30

sh.	ct.	sh.	ct.
2	45	3	10
–	95	–	55

Make at least 10 more examples based on these.

Step 3. Subtraction of shillings and cents (using 5 cents) from shillings and cents without carrying from shillings column.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
1	25	1	75	1	60	2	60
– 1	15	– 1	30	– 1	35	– 1	50

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
3	15	8	20	10	40	15	95
– 1	05	– 5	15	– 7	25	– 12	35

Make more examples of this type, gradually increasing to sh. 99.99 in the top line.

Step 4. Subtraction of shillings and cents (5 cents) from shillings and cents, carrying from the shilling column.

sh.	ct.	sh.	ct.	sh.	ct.	sh.	ct.
2	55	3	20	6	80	10	05
– 1	65	– 1	35	– 3	95	– 7	10

sh.	ct.	sh.	ct.
15	00	19	60
– 13	70	– 15	85

Make 20 more examples based on the above, gradually increasing to sh. 100 in the top line.

Step 5. The use of 1 cent pieces should now be introduced.

sh.	ct.	sh.	ct.	sh.	ct.	
15	37	36	42	78	63	etc.
− 9	59	−21	60	−69	95	

The maximum top number should be sh. 99.99.

MONEY

MULTIPLICATION

Aim

Short multiplication of money by numbers up to 12; the answer not to exceed sh. 99.99.

Gradually increase the numbers, giving the children plenty of practice.

Introduce 5 cent pieces first, then 1 cent pieces.

Step 1.

sh.	ct.	
4	15	
	×2	
8	30	*Ans.*

sh.	ct.	
3	35	
	×4	
13	40	*Ans.*
1	140=1.40	
12		
13		

sh.	ct.	
6	75	
	×7	
47	25	*Ans.*
5	525=5.25	
42		
47		

sh.	ct.	
7	25	
	×9	
65	25	*Ans.*
2	225=2.25	
63		
65		

sh.	ct.	
18	05	
	×5	
90	25	*Ans.*

sh.	ct.	
22	65	
	×3	
67	95	*Ans.*
1	195=1.95	
66		
67		

sh.	ct.	
6	85	
	×12	
82	20	*Ans.*
10	1020=10.20	
72		
82		

Make 20 more examples on these lines.

Step 2. Introduce 1 cent pieces.

sh.	ct.		sh.	ct.		sh.	ct.	
3	22		5	64		9	57	
	×3			×6			×7	
9	66	*Ans.*	33	84	*Ans.*	66	99	*Ans.*
			3	384=3.84		3	399=3.99	
			30			63		
			33			66		

sh.	ct.		sh.	ct.		
23	41		7	29		
	×4			×12		etc.
93	64	*Ans.*	87	48	*Ans.*	
1	164=1.64		3	348=3.48		
92			84			
93			87			

Make 10 more examples, to give children plenty of practice.

DIVISION

Aim

Short division within known tables, the dividend not to exceed sh. 99.99. No remainder should be introduced at first.

Step 1. Division of cents only, without a remainder, first giving only cents of 10 cent value, then 5 cent and lastly single cent:

	sh.	ct.	sh.	ct.	sh.	ct.	
(*a*)	3)	30	4)	80	5)	70	
		10		20		14	
(*b*)	5)	55	3)	45	9)	45	etc.
(*c*)	2)	82	6)	96	3)	51	
	12)	72	9)	63			etc.
		06		07			

Step 2. Division of shillings and cents, without a remainder. Note that stage (*a*) does not involve a transfer of shillings into cents.

	sh.	ct.	sh.	ct.	sh.	ct.
(*a*)	2)2	20	3)6	90	4)8	80
	sh.	ct.	sh.	ct.		
	7)7	77	6)24	42	etc.	

More examples to be made by the teacher.

(*b*) Introduce the carrying of a shilling, and at first insert the carried figure. Make sure again that the children understand the principle involved in the carrying of a shilling; that it is *not* carrying a 1, but carrying and changing it into 100 ct.

sh.	ct.		sh.	ct.		sh.	ct.	
	1			2			2	
10)1	00		8)2	40		9)2	70	
	10	*Ans.*		30	*Ans.*		30	*Ans.*
sh.	ct.		sh.	ct.		sh.	ct.	
	4 2			6			1 1	
5)24	75		7)48	37		3)94	68	
4	95	*Ans.*	6	91	*Ans.*	31	56	*Ans.*

Give many more examples.

Step 3. Division of cents only, with a remainder, treating as in Step 1. Note the writing of ct. in the remainder.

sh. ct.	sh. ct.	sh. ct.
3) 80	6) 40	4) 75
26 r 2 ct. *Ans.*	06 r 4 ct. *Ans.*	18 r 3 ct. *Ans.*
sh. ct.	sh. ct.	sh. ct.
5) 57	6) 43	9) 94
11 r 2 ct. *Ans.*	07 r 1 ct. *Ans.*	10 r 4 ct. *Ans.*

Step 4. Division of shillings and cents, with a remainder:

(*a*) No carrying from sh. to ct.

sh.	ct.	sh.	ct.	sh.	ct.
	2				
4)4	67	4)16	45	9)27	84
1	16 r 3 ct. *Ans.*	4	11 r 1 ct. *Ans.*	3	09 r 3 ct. *Ans.*

(*b*) Carrying from sh. to ct. Insert carrying figure.

```
sh.  ct.            sh.  ct.              sh.  ct.
     3 4                 7 5                    4
7)45  24           12)91  78               6)76  21
  6   46 r 2 ct. Ans.  7  64 r 10 ct. Ans.  12   70 r 1 ct. Ans.
```

More examples to be made by teacher, gradually increasing number in shillings column of dividend to 99.

LINEAR MEASURE

Aim

Simple 4 rules in yd. ft. and then in yd. ft. in. having yd. in the answer only.

ADDITION—YARDS AND FEET

Step 1. Begin by adding feet to feet, giving the answer in yards and feet. This stage will be *short* and its chief aim is to establish the method of setting down and working.

(i) *Teacher:* ' When we want to write the words " yards " and " feet ", we write a short form of these words, like this :

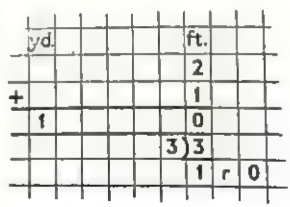
yd. ft.

When you are writing them in your books put " yd." in one square : then leave five empty squares : then put " ft." in one square : like this :

yd. ft. (Demonstrate on blackboard)

(ii) ' Now we want to add together 2 feet and 1 foot. What must we write first? (The answer required is " yd. in one square, five squares empty, ft. in ano ther square")

Then we put down the sum like this ' :



At each point explain what happens, thus :

' If I add 2 and 1 my answer is 3.' (Put 3 under the answer space as shown.) Now what are those 3 things? And what is another name for 3 feet? ' (' It is 1 yard.') ' Now, remember always—since 3 feet = 1 yard, we cannot have more than

2 feet in the " feet " column of a sum in yards and feet. But we want to know how to find out how many yards we have got in our answer. We do this by dividing the number of feet by three. This is how you do it in your books—put down the answer to $3 \div 3$ like an ordinary division sum. Now—our answer is 1. 1 what?' ('1 yard.') 'Are there any feet left over?' ('No.') 'So we write here " r 0 ". Next we put the remainder in the " feet " column, and the answer to the division *under* the answer line of the yards column, until we see if there are any more yards to add. There are none—so we put the figure 1 in its proper place on the answer line.'

(iii) Now let the pupils work 1 ft. + 2 ft.; and 2 ft. + 2 ft. to establish the method, the teacher guiding them individually.

Step 2. When the method of setting down and working has been thoroughly grasped (this should only need one or two periods for the average pupil, though the slower ones may need rather more time), it is time to introduce sums with yards and feet in one item and feet in the other, as follows:

(*a*) Demonstrate *first* a sum like this:

$$
\begin{array}{rr l}
\text{yd.} & \text{ft.} & \\
4 & 2 & \\
+\quad & 2 & \\
\hline
5 & 1 & \textit{Ans.} \\
\hline
1 & 3\underline{)4} & \\
\underline{4} & 1 \text{ r } 1 & \\
5 & & \\
\end{array}
$$

(*b*) Next demonstrate a sum in which there are feet in the first line, yards and feet in the second line.

$$
\begin{array}{rr l}
\text{yd.} & \text{ft.} & \\
 & 2 & \\
5 & 1 & \\
\hline
6 & 0 & \textit{Ans.} \\
\hline
1 & 3\underline{)3} & \\
\underline{5} & 1 \text{ r } 0 & \\
6 & & \\
\end{array}
$$

Explain each point issuing the same kind of explanation as that on previous page. It is important to say :

' In a sum in yards and feet, if we have no *yards*, we leave the space *empty*. We do not write ' :

yd.
0

But in a sum in yards and feet we always *write* the 0 if we wish to say ' no feet '.

The quantity ' two yards and no feet ' is sometimes seen as :

yd.	ft.
2	—

This is never to be done in schools. The quantity must *always* be written, e.g.

yd.	ft.
2	0

(*c*) Now make for the class at least ten sums like Example 1, and at least 10 like Example 2. Here are some ideas :

	yd.	ft.	yd.	ft.	yd.	ft.	yd.	ft.
Ex. 1.	4	0	5	1	2	2	3	2
	+	2	+	1	+	1	+	2

	yd.	ft.	yd.	ft.	yd.	ft.	yd.	ft.
Ex. 2.		2		0		1		2
	+4	2	+3	0	+5	2	+4	0

Work on this type of sum should not take more than one or two periods provided the class are given enough time in that one or two periods to do *at least* ten examples of Exercise 1 and ten of Exercise 2 *by themselves.*

Step 3. Then demonstrate a sum like this.

yd.	ft.	
4	2	
+3	1	
8	0	*Ans.*
1	3)3	
7	1 r 0	
8		

As before, make up at least 10 more examples for the class to work by themselves.

yd.	ft.	yd.	ft.	yd.	ft.	yd.	ft.
4	2	3	1	9	0	5	2
+5	1	+6	0	+4	2	+3	2

This section again should *not* need more than 1 or 2 periods, but children must do at least 10 examples by themselves.

Step 4. Lastly demonstrate a sum like this, with 3 items.

yd.	ft.	
4	2	
3	2	
+5	2	
14	0	*Ans.*
2	3)6	
12	2 r 0	
14		

Make at least ten more examples of your own for the class to work by themselves. When making your own examples, no sum total may be greater than 21 yd. 2 ft.

SUBTRACTION—YARDS AND FEET

The form of setting out is as in addition.

Step 1. Subtraction without carrying. Check knowledge of '1 yd. = 3 ft.'

Revise subtraction of number in Mental Arithmetic.

Then :

	yd.	ft.			yd.	ft.	
(i)	4	2		(ii)	3	2	
	−	1			−1	0	
	4	1	*Ans.*		2	2	*Ans.*

(*a*) Do No. 1 on blackboard, children doing the working.

(*b*) Revise the empty space for no yards but a zero for no feet in yards and feet sums.

(*c*) Children should say ‘Two feet take away one foot equals one foot. Write one foot in the answer line. Four yards take away no yards equals four yards. Write four yards in the answer line’. Then do No. 2 on blackboard in the same way. Then let children do these sums in their books:

yd.	ft.	yd.	ft.	yd.	ft.	yd.	ft.
4	1	3	2	2	2	10	1
–	1	–	1	–	2	–	1

yd.	ft.	yd.	ft.	yd.	ft.	yd.	ft.
9	2	4	2	5	1	2	2
–	1	–2	0	–3	0	–1	0

yd.	ft.	yd.	ft.
2	0	10	0
–1	0	– 8	0

This work is sufficient for *only* about one period.

Step 2. Subtraction with a carrying figure from yd. to ft.

A first blackboard example should be done as follows:

yd.	ft.	
9	4	
~~10~~	~~1~~	
– 5	2	
4	2	*Ans.*

Say ‘One foot take away two feet I cannot do, so I take a yard from my ten yards in the top line, leaving nine, change the yard into feet. How many feet in a yard? (3) Three feet and one foot make four feet. So I have nine yards and four feet. Four feet take away two feet leaves two feet. So I write 2 feet in the answer line. Nine yards take away five yards leaves four yards’.

Give at least twenty examples of your own, like these:

yd.	ft.	yd.	ft.	yd.	ft.	yd.	ft.
4	1	10	0	9	1	8	0
–3	2	– 7	1	–4	2	–5	2

SHORT MULTIPLICATION—YARDS AND FEET

Revise what must be done if we have an answer of more than 2 feet in a yards and feet sum.

Then proceed in steps, demonstrating about 3 sums in each step and give the class at least 10 examples to do at the end of each step.

Step 1.

yd.	ft.	
	2	
	×9	
6	0	*Ans.*
6	3)18	
	6 r 0	

yd.	ft.	
	2	
	×8	
5	1	*Ans.*
5	3)16	
	5 r 1	

Step 2.

yd.	ft.	
2	1	
	×9	
21	0	*Ans.*
3	3)9	
18	3 r 0	
21		

yd.	ft.	
3	2	
	×5	
8	1	*Ans.*
3	3)10	
15	3 r 1	
18		

yd.	ft.	
3	0	
	×7	
21	0	*Ans.*

Note carefully:

When you make up your own examples be sure that the answer is never more than 21 yards 2 feet. Two periods of practice, if the class do at least five sums in Step 1 and twenty in Step 2, should be sufficient.

SHORT DIVISION—YARDS AND FEET

After the class has grasped the method of short multiplication, short division is easily understood.

Step 1. Short.

$$\begin{array}{r|rr} & \text{yd.} & \text{ft.} \\ 4) & 4 & 0 \\ \hline & 1 & 0 \end{array}\ \textit{Ans.} \qquad \begin{array}{r|rr} & \text{yd.} & \text{ft.} \\ 2) & 4 & 2 \\ \hline & 2 & 1 \end{array}\ \textit{Ans.}$$

This step is only intended to demonstrate the layout of a sum, and no examples need be worked by the class.

Step 2. Division with *no* remainder. When doing blackboard demonstration revise the method of combining ' carried yards ' with feet.

$$\begin{array}{r|rr} & \text{yd.} & \text{ft.} \\ & & 10 \\ 5) & 18 & \cancel{1} \\ \hline & 3 & 2 \end{array}\quad \textit{Ans.}$$

Say ' Five into eighteen goes three times and three yards over. Put 3 yards in the answer line. Three yards equals nine feet. Nine and one are ten. Cross out the one and write 10 to the right of it. Five into ten goes twice. Put 2 feet in the answer. Answer 3 yards and two feet '.

Then make additional examples—not less than ten—for the class to work, like this :

$$\begin{array}{r|rr} & \text{yd.} & \text{ft.} \\ 7) & 16 & 1 \\ \hline \end{array} \qquad \begin{array}{r|rr} & \text{yd.} & \text{ft.} \\ 8) & 13 & 1 \\ \hline \end{array} \qquad \begin{array}{r|rr} & \text{yd.} & \text{ft.} \\ 2) & 19 & 1 \\ \hline \end{array}$$

Step 3. Division with no remainders and with no yards in answer.

Demonstrate one example on blackboard, at the same time revising ' change yards to feet, cross out the feet and put new figure on the right ', and no 0 under yards.

$$\begin{array}{r|rr} & \text{yd.} & \text{ft.} \\ 7) & 4 & \cancel{2}\ 14 \\ \hline & & 2 \end{array}\ \textit{Ans.}$$

Say, ' Seven into four will not go : so I have no yards in my answer. Leave an empty space under the yards. Four yards equals twelve feet : twelve plus two equals fourteen. Cross out the two feet and put 14 on the right of it. 7 into 14 goes twice. Put two feet in the answer. Answer 2 feet.'

Then make up at least ten examples for the class to do by themselves, like this:

yd. ft.
5)3 1

yd. ft.
11)7 1

yd. ft.
10)3 1

Step 4. Division with remainder.

(*a*) No carrying from yards to feet. A short stage, five examples are enough for children to do in their books. In this step revise meaning of ' r ' and need to write in what the remaining quantity is: i.e. not simply 2 but 2 ft.

yd. ft.
5)15 2
3 0 r 2 ft. *Ans.*

yd. ft.
4)16 1
4 0 r 1 ft. *Ans.*

(*b*) Carrying from yards to feet.

yd. ft.
6)20 ~~1~~ 7
3 1 r 1 ft. *Ans.*

yd. ft.
9)21 ~~2~~ 11
2 1 r 2 ft. *Ans.*

yd. ft.
7)16 ~~2~~ 8
2 1 r 1 ft. *Ans.*

yd. ft.
8)13 ~~2~~ 17
1 2 r 1 ft. *Ans.*

yd. ft.
4)9 ~~2~~ 5
2 1 r 1 ft. *Ans.*

yd. ft.
3)10 ~~2~~ 5
3 1 r 2 ft. *Ans.*

yd. ft.
10)13 ~~2~~ 11
1 1 r 1 ft. *Ans.*

yd. ft.
5)18 ~~2~~ 11
3 2 r 1 ft. *Ans.*

yd. ft.
4)14 ~~0~~ 6
3 1 r 2 ft. *Ans.*

yd. ft.
5)17 ~~0~~ 6
3 1 r 1 ft. *Ans.*

Note carefully:

The remainder can never be more than 2 feet in these examples. Teachers making their own examples should be extremely careful to observe this rule.

(*c*) Division with no yards in answer and remainder.

yd. ft. 4)2 $\cancel{0}$ 6 1 r 2 ft. *Ans.*	yd. ft. 6)4 $\cancel{2}$ 14 2 r 2 ft. *Ans.*
yd. ft. 7)5 $\cancel{1}$ 16 2 r 2 ft. *Ans.*	yd. ft. 12)8 $\cancel{2}$ 26 2 r 2 ft. *Ans.*
yd. ft. 9)6 $\cancel{1}$ 19 2 r 1 ft. *Ans.*	yd. ft. 8)5 $\cancel{2}$ 17 2 r 1 ft. *Ans.*
yd. ft. 11)7 $\cancel{2}$ 23 2 r 1 ft. *Ans.*	yd. ft. 5)3 $\cancel{2}$ 11 2 r 1 ft. *Ans.*
yd. ft. 11)4 $\cancel{0}$ 12 1 r 1 ft. *Ans.*	yd. ft. 9)3 $\cancel{1}$ 10 1 r 1 ft. *Ans.*

ADDITION OF FEET AND INCHES

Step 1. Refer to the foot ruler which each boy and girl should have and say, 'We have been doing sums in yards and feet. But you learned in Class 2 that sometimes we want to measure something which is even smaller than one foot. Your ruler is one foot long but it is divided into twelve parts. What are these parts called? How many inches are there in a foot? Yes, there are twelve.

'Now, we have our big measure called yards. (Put yards on blackboard) then the not very big measure called feet (on blackboard) and our small measure called inches (on blackboard).

'We learned that for yards we write "yd." (on blackboard), for feet we write "ft.", and so for inches we only write a short word. This is it—"in."' (on blackboard).

(Your blackboard should now look like this:

yards	feet	inches
yd.	ft.	in.)

'Now for the next few lessons we will talk about feet and inches only, so I can rub yards off the board.' (Do so.)

Here revise rule for setting down (ft. in one square, leave five empty squares, in. in one square). Then, using the same kind

of explanation as for yds. and feet, go on to the first sums: and then to Step 2, etc.

(*a*) Simple setting out practice.

	ft.	in.	
		4	
+		7	
		11	*Ans.*

	ft.	in.	
		5	
+		4	
		9	*Ans.*

N.B. There is no 'changing into feet' here: but you may be asked, 'Why do you not put a 0 in the feet column?' You remind them that we put a 0 in the feet column when the sum was yards and feet. *In feet and inches, the rule is 'Leave an empty space in the feet column and always put a 0 in the inches column'.*

(*b*) Addition of inches with answers in feet and inches (i.e. changing inches to feet). Refer back to notes on changing feet to yards, and explain change of inches to feet (and 'only 11 inches in the inches column') in the same way. Then:

	ft.	in.	
		9	
+		4	
	1	1	*Ans.*
	1	12)13	
		1 r 1	

	ft.	in.	
		7	
+		5	
	1	0	*Ans.*
		12)12	
		1 r 0	

	ft.	in.	
		8	
+		6	
	1	2	*Ans.*
	1	12)14	
		1 r 2	

Now work out at least 20 examples like these for the class to do.
N.B. There must be no feet except in the answer.

Step 2. This step introduces ft. in the top, then in the bottom, line only; both must be demonstrated but they can be done fairly quickly.

(*a*)

	ft.	in.	
	1	9	
+		5	
	2	2	*Ans.*
	1	12)14	
	1	1 r 2	
	2		

(*b*)

	ft.	in.	
		9	
+	1	6	
	2	3	*Ans.*
	1	12)15	
	1	1 r 3	
	2		

Now give the class twenty examples—ten of (*a*) and ten of (*b*). The slower children *may* require further work. Notice that in one line there are feet and inches, in the other only inches. The total must *not* be more than 2 ft. 11 in.

Step 3. This stage consists of a short introduction (preferably done by questioning the class) leading to sums with *yards in the answer only.* Give the class these sums on the blackboard :

(*a*)

	ft.	in.	
	1	5	
+	1	6	
	2	11	*Ans.*

(*b*)

	ft.	in.	
	1	6	
+	1	6	
		0	*Ans.*
	1	12)12	
	2	1 r 0	
	3		

(i) When sum number (*b*) has reached the stage shown, question the class about the rule '2 ft. only in the ft. column': and question until a child suggests that we must 'change it into yards'. Then write the abbreviation 'yd.' to the left of 'ft.' and complete the sum as shown below :

	yd.	ft.	in.	
		1	6	
+		1	6	
	1	0	0	*Ans.*
		1	12)12	
		2	1 r 0	
		3)3		
		1 r 0		

(Don't forget to move the +)

(ii) Now stress the rule of 'five empty spaces between yd. ft. in.': and in future set all sums out *as* yd. ft. in.

(iii) Revise again the rule for 'putting a 0': but this time state it in its final form : 'Always put a 0 where there is another figure to be written on the left.'

(iv) The children should now work *twenty sums* on their own—ten like (*a*) five like (*b*), and five like (*c*) below.

Examples:

(*a*)

	yd.	ft.	in.	
		2	8	
+			9	
	1	0	5	*Ans.*
	1	1	12)17	
		2	1 r 5	
		3)3		
		3 r 0		

(*b*)

	yd.	ft.	in.	
			10	
+		2	6	
	1	0	4	*Ans.*
		1	12)16	
		2	1 r 4	
		3)3		
		1 r 0		

(*c*)

	yd.	ft.	in.
		2	9
+		2	5
	1	2	2
	1	1	12)14
		4	1 r 2
		3)5	
		1 r 2	

Step 4. Introduce sums with 3 items. Do two on the blackboard, and give an exercise of about ten sums like them. Do *not* in any item have more than 2 ft. Although it is common to use feet in numbers over 2, such as for the height of a man, it is not good at this point to drill that every 3 ft. make a yard and then confuse children by allowing expressions of 5 ft. on occasions.

ft.	in.	ft.	in.
2	2	1	8
1	11	2	6
+1	6	+2	9

SUBTRACTION—FEET AND INCHES

Step 1. Revise: 3 ft. = 1 yd., 12 in. = 1 ft. In mental arithmetic revise subtraction of pure number *and* yards and feet.

Also in mental arithmetic introduce 'inches minus inches'—*no* feet at all. Then :

$$
(a)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ & 11 \\ - & 3 \\ \hline & 8 \\ \hline \end{array}\ \textit{Ans.}
\qquad
(b)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 1 & 10 \\ - & 9 \\ \hline 1 & 1 \\ \hline \end{array}\ \textit{Ans.}
\qquad
(c)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 1 & 9 \\ -1 & 8 \\ \hline & 1 \\ \hline \end{array}\ \textit{Ans.}
$$

These three sections can be covered in one period. Step (*a*) may be omitted for the whole class, and used for backward boys only. Steps (*b*) and (*c*) should be demonstrated, and the class should then do not less than ten examples.

$$
(d)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 2 & 8 \\ -1 & 4 \\ \hline 1 & 4 \\ \hline \end{array}\ \textit{Ans.}
$$

Make ten more examples of section (*d*). The rules—2 ft. in top line, 1 ft. in bottom line, no carrying ft. to in.

Step 2. Introduce 'carrying figure' from feet to inches. Revise rule for deletion of original inches figure.

Demonstrate

$$
(a)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 1 & 21 \\ \cancel{2} & \cancel{9} \\ -1 & 10 \\ \hline & 11 \\ \hline \end{array}\ \textit{Ans.}
\qquad
(b)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 1 & 16 \\ \cancel{2} & \cancel{4} \\ & 9 \\ \hline 1 & 7 \\ \hline \end{array}\ \textit{Ans.}
$$

Make up at least ten examples of (*a*) and ten of (*b*) for the class to do. N.B. Never have more than 2 ft. : make sure that there are more inches in the bottom line than in the top : and always put 0 when necessary in the inches column, *never* a dash (—).

$$
(a)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 2 & 2 \\ -1 & 9 \\ \hline & \\ \hline \end{array}
\qquad
(b)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 2 & 4 \\ - & 10 \\ \hline & \\ \hline \end{array}
\qquad
(c)\quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 1 & 4 \\ - & 11 \\ \hline & \\ \hline \end{array}
$$

SHORT MULTIPLICATION—FEET AND INCHES

Step 1. Revise rule of setting down and changing inches into feet. Then follow the same kind of explanation as in Step 1 of yards and feet (page 31). Work this example on the blackboard

ft.	in.	
	9	
	$\times 3$	
2	3	*Ans.*
2	12)27	
	2 r 3	

The class should now work the following examples: 11 in. × 3, 10 in. × 3, 9 in. × 3, 8 in. × 4, 7 in. × 5, 7 in. × 4, 6 in. × 5, 5 in. × 7, 5 in. × 6, 3 in. × 9, 4 in. × 8, 3 in. × 10, 10 in. × 2, 11 in. × 2, 9 in. × 2.

Step 2. Revise the explanation of changing to yards given in 'Addition, Step 3 (*b*)' (page 41). Revise setting out of yd. ft. in. Revise rule for 'putting in the 0'. Then:

	yd.	ft.	in.	
(*a*)			11	
			$\times 4$	
	1	0	8	*Ans.*
		3)3	12)44	
		1 r 0	3 r 8	

Examples for class to do: 9 in. × 5, 4 in. × 10, 8 in. × 5, 10 in. × 4, 7 in. × 6, 5 in. × 9, 6 in. × 7, 4 in. × 11, 3 in. × 12.

	yd.	ft.	in.	
(*b*)		2	11	
			$\times 12$	
	11	2	0	*Ans.*
	11	11	12)132	
		24	11 r 0	
		3)35		
		11 r 2		

This is the *biggest* measure and biggest multiplier that will be used in Class 3. Make at least twenty examples for the class to work. Remember, the top line must *not* be more than 2 ft. 11 in. and the multiplier *not* more than 12.

SHORT DIVISION—FEET AND INCHES

Step 1. A very short stage to establish method and layout.

$$\begin{array}{rll} & \text{ft.} & \text{in.} \\ 2) & 2 & 10 \\ \hline & 1 & 5 \ \textit{Ans.} \\ \hline \end{array}$$

The only possible divisor at this stage is 2, since one can never have more than 2 ft. and we need ft. and in. in the answer. Let the class quickly do:

$$\begin{array}{rll} & \text{ft.} & \text{in.} \\ 2) & 2 & 8 \\ \hline \end{array} \qquad \begin{array}{rll} & \text{ft.} & \text{in.} \\ 2) & 2 & 6 \\ \hline \end{array} \qquad \begin{array}{rll} & \text{ft.} & \text{in.} \\ 2) & 2 & 4 \\ \hline \end{array} \qquad \begin{array}{rll} & \text{ft.} & \text{in.} \\ 2) & 2 & 2 \\ \hline \end{array}$$

Step 2. Answers with no remainder carrying feet to inches. Revise 12 in. = 1 ft. and method of carrying. Changing feet to inches must be mental.

$$\begin{array}{rll} & \text{ft.} & \text{in.} \\ & & 32 \\ 8) & 2 & \cancel{8} \\ \hline & & 4 \quad \textit{Ans.} \\ \hline \end{array}$$

Class now do twenty examples like these:

$$\begin{array}{rll} & \text{ft.} & \text{in.} \\ 11) & 2 & 9 \\ \hline \end{array} \qquad \begin{array}{rll} & \text{ft.} & \text{in.} \\ 10) & 2 & 6 \\ \hline \end{array} \qquad \begin{array}{rll} & \text{ft.} & \text{in.} \\ 9) & 2 & 3 \\ \hline \end{array} \qquad \begin{array}{rll} & \text{ft.} & \text{in.} \\ 12) & 2 & 0 \\ \hline \end{array} \qquad \begin{array}{rll} & \text{ft.} & \text{in.} \\ 2) & 1 & 10 \\ \hline \end{array}$$

Step 3. Division including changing feet to inches with remainder. In explanation stress the putting of 'in.' after the remainder.

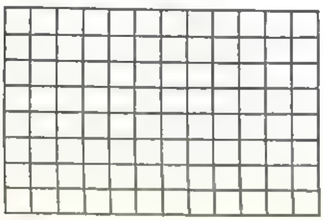
$$\begin{array}{rll} & \text{ft.} & \text{in.} \\ & & 35 \\ 8) & 2 & \cancel{11} \\ \hline & & 4 \text{ r } 3 \text{ in.} \ \textit{Ans.} \\ \hline \end{array}$$

Now work out twenty similar examples for class to do.

FRACTIONS

Equipment. First of all make or obtain a series of pieces of paper, all of identically the same shape and size. There should be at least two for every child in the class and several spare ones as well.

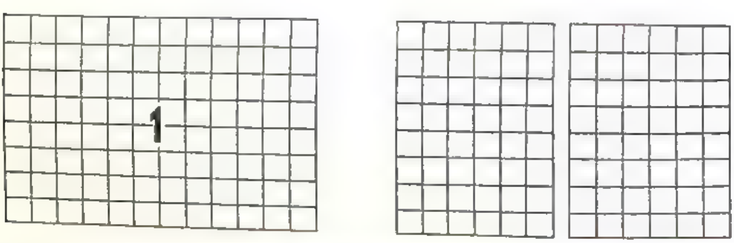
These pieces of paper may be either circular or rectangular and may be cut from newspaper, writing paper or any other material available. If available it is best to obtain rectangular pieces of squared paper from a normal arithmetic exercise book, say 12 squares long and 8 squares deep.



The teacher must also have similar shaped rectangles marked off in the same number of squares, but these should be much larger and made of rather stiffer material so that they can easily be seen from the back of the class.

Step 1 (1 lesson). Introduction to the terms 'half' and 'halves'.

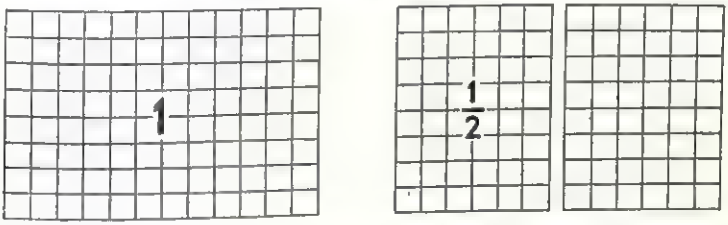
(*a*) Each child takes his 2 pieces of paper; on the first he writes the figure 1, while the other is torn or cut neatly into halves as shown:



(*b*) The first piece is then compared with the two smaller pieces and gradually the answers that the first is 'a whole one' and that the others are 'two halves' are elicited. The children will

probably not provide the answers in so many words (especially as some vernaculars have no accurate terms for exact fractions) but when the children have expressed a general understanding of the relationship of the size of the two smaller pieces to the large one the teacher explains and writes up on the blackboard the terms $\frac{1}{2}$ (half) and 1 (whole one, or unit).

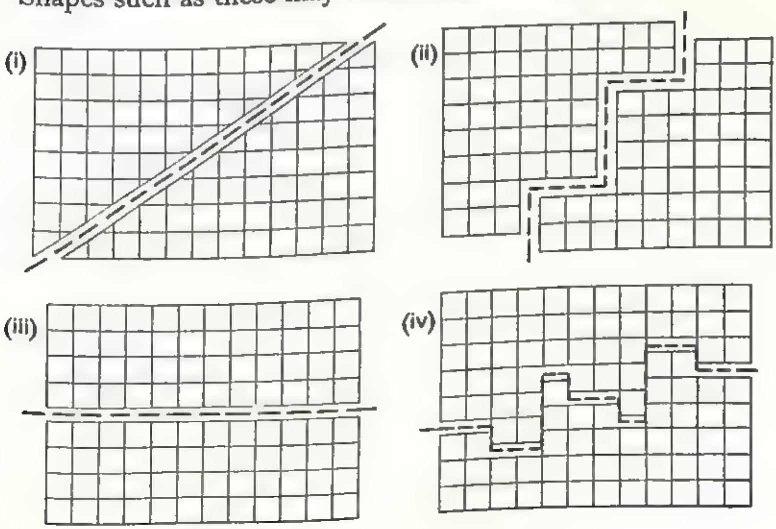
(*c*) As the child has already written '1' on the whole, they can now go on (having understood its meaning) to write '$\frac{1}{2}$' on one of the two smaller pieces of paper, as follows:



(The other half may be left blank, as the children can see that it is the same size as the first half, and it will be required for other purposes anyway later on.)

(*d*) The children now form into small groups and each group is given several of the extra rectangles of paper and is asked to cut each into as many pairs of equal halves of different shapes as it can.

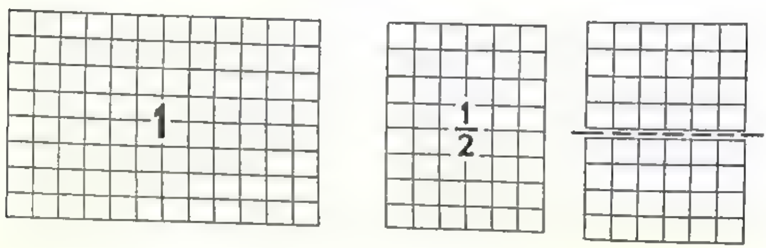
Shapes such as these may then be obtained:



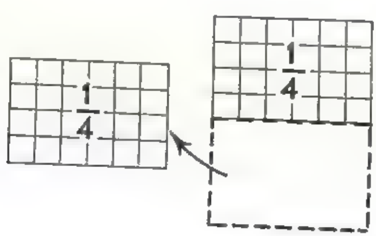
N.B. As far as possible the teacher should leave the children to discover these fractional shapes by themselves. He might put one up on the blackboard to start with (e.g. No. (ii) above) but not more than one.

Step 2 (1 lesson). Introduction to quarters.

(*a*) This lesson follows the same pattern as the previous one. Each child has his 3 pieces of paper to start with:



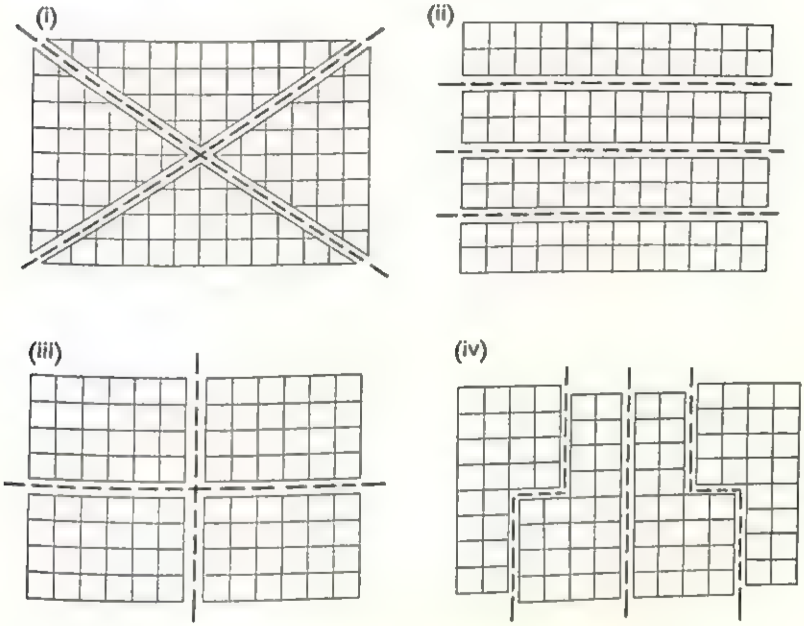
The third unmarked piece is taken and again cut into two equal pieces. This leads after questioning to the introduction of the term $\frac{1}{4}$ (quarter) and it is then marked upon these two smallest pieces of paper by each child.



(*b*) The teacher must then ensure that it is clearly understood that 4 quarters make a whole. This can easily be done if the children work in pairs and put their 4 quarters in such a position that they cover up exactly one of the unit pieces.

(*c*) When step (*b*) is clearly understood, and only then, the children divide into groups again and, working on extra pieces

of paper, try to find as many different shaped quarters as possible.



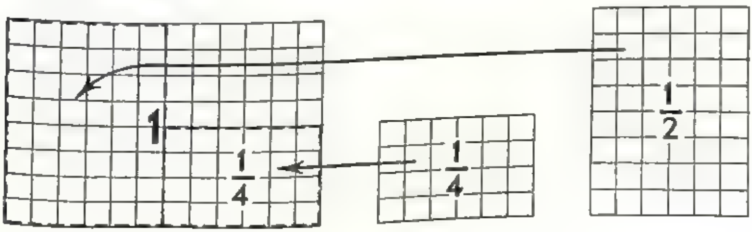
Step 3. Relationships of $\frac{1}{4}$s and $\frac{1}{2}$s and units.

If 2 (*c*) above has been completed properly it should soon be possible to elicit that two quarters, no matter what their shape, always fit together to make a half (as in 1 (*d*) above). If this is clearly understood the more formal work which follows is quickly grasped. If not, it is essential to go back to steps 1 (*d*) and 2 (*c*) again.

Equally two halves (or 4 quarters) always fit together to make a whole.

Step 4. Simple fractional addition and subtraction.

(*a*) The children take their four separate pieces of paper and by simply placing and removing the appropriate pieces of paper



from on top of each other the correct answers to the following questions are easily seen:

(i) $\frac{1}{4}+\frac{1}{4}$ (ii) $\frac{1}{2}+\frac{1}{2}$

N.B. They already know that $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ or that 2 quarters equal a half.

(iii) $1-\frac{1}{2}$; (iv) $\frac{1}{2}-\frac{1}{4}$.

(*b*) Then we come to

$$\frac{1}{2}+\frac{1}{4}$$

which introduces the new idea of $\frac{3}{4}$. If the previous work has been properly understood this concept of three-quarters will quickly be grasped.

It should be noted that at this stage it is *not necessary* to introduce the formal notation of

$$\frac{1}{2}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$$

The children are working with their visual aids and it is quite sufficient for them simply to write

$$\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

(*c*) Then follow other similar examples which the teacher writes on the blackboard. The children work these practically, writing *only* the answers in their books.

(i) $1-\frac{1}{4}$, $\frac{1}{4}+\frac{1}{2}$, $1-\frac{1}{2}$, $1-\frac{3}{4}$

leading quickly to

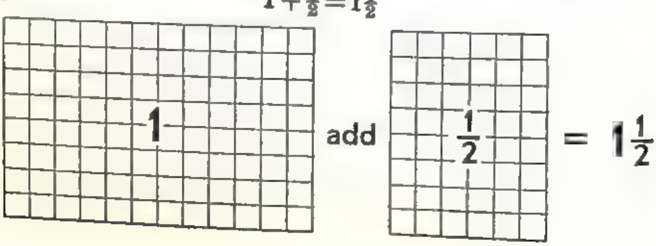
(ii) $\frac{3}{4}-\frac{1}{2}$, $\frac{3}{4}+\frac{1}{4}$, $\frac{3}{4}-\frac{1}{4}$, $\frac{1}{4}+\frac{3}{4}$

Step 5.

(*a*) There is no reason why these examples should not be continued to deal with terms greater than one. The teacher, however, must remember that at this stage the work is still very informal, and he must *not* start to use such terms as 'mixed fractions'. It is sufficient simply to take the example:

$1+\frac{1}{2}$ and explain that we can write this down as $1\frac{1}{2}$.

Thus $1+\frac{1}{2}=1\frac{1}{2}$



(*b*) This can then be followed by simple sums to be worked out by the children moving gradually through these stages (i–iv), writing only the answers, as in 4 (*c*).

(i) $1+\frac{1}{4}$; $1+\frac{3}{4}$; $1+1$.

(ii) $2-1$; $2-\frac{3}{4}$; $2-\frac{1}{4}$; $2-\frac{1}{2}$.

(iii) $\frac{3}{4}+\frac{1}{2}$; $1\frac{1}{4}+\frac{1}{4}$; $\frac{1}{2}+\frac{3}{4}$; $1+\frac{1}{4}$; $1\frac{1}{4}+\frac{1}{2}$; $\frac{3}{4}+\frac{3}{4}$.

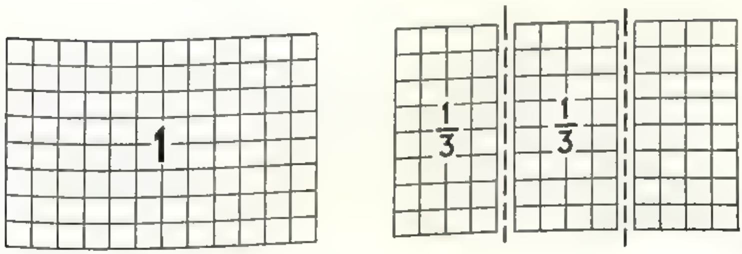
(iv) $1\frac{1}{4}-\frac{1}{2}$; $1\frac{1}{2}-\frac{1}{2}$; $1\frac{1}{2}-\frac{3}{4}$; $1\frac{1}{4}-\frac{3}{4}$; $1\frac{1}{4}-1$.

(v) These can then be followed by a whole series of similar mixed questions to make the children really familiar with these fractions. In making up questions the teacher should not exceed the unit 2 either in question or answer, (but it is quite safe to go up to this total as the child has two complete visual units available).

Step 6. Practical introduction of thirds and sixths.

The children are each given 1 fresh 'unit' piece of paper and then the teacher and class together go through the stages outlined for halves in steps 1 (*a*)–(*c*) and for quarters in 2 (*a*) and 2 (*b*) above, using the same whole unit piece as they have used previously.

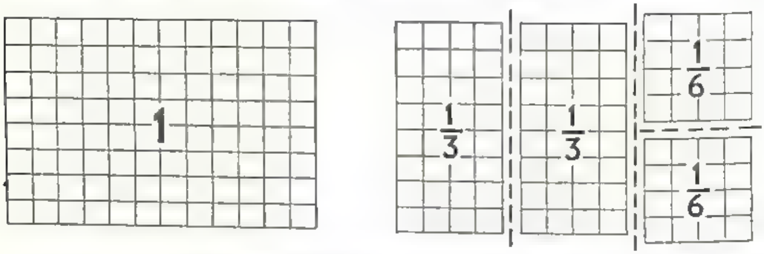
(*a*) The unit is left uncut, while the other piece is divided this time into thirds.



(*b*) When the children clearly understand that the 3 smaller pieces together make up a whole unit, two of the pieces should be marked as thirds ($\frac{1}{3}$).

(*c*) As the children have already done the same for halves and

quarters it should be possible to proceed quickly to the halving of the final third to make two 'one-sixth' pieces:

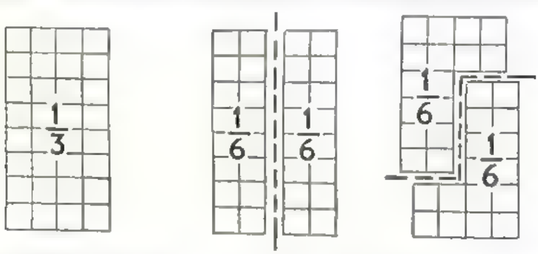


and also to the understanding of the facts that three thirds make one whole unit, two sixths make a third, and six sixths make one whole unit. All of this is done orally, of course. (For this last point the children can work together in threes so that they will have enough $\frac{1}{6}$ pieces to make a whole unit.)

Note carefully:

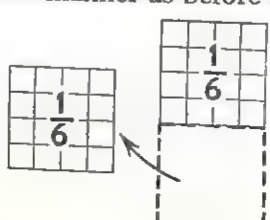
Having already determined by practical experiments that two quarters always make a half it should be possible to establish the similar connection between sixths and thirds purely theoretically without having to do any practical work.

For backward children it may, however, be necessary to form a small group and let them re-discover this fact as follows:



Step 7. Simple fractional addition and subtraction of thirds and sixths.

Again this is done with the children using their pieces of paper (5 this time) in the same manner as before:



The same stages are followed and once again teachers should notice that the working is informal and the children merely put

$$\tfrac{1}{3}+\tfrac{1}{6}=\tfrac{3}{6}$$

If any child is bright enough to discover for himself the fact that

$$\tfrac{1}{3}+\tfrac{1}{6}=\tfrac{3}{6}=\tfrac{1}{2}$$

congratulate him but do not attempt to go into a general explanation of this new idea with the whole class and *on no account* introduce the formal

$$\frac{1}{3}+\frac{1}{6}=\frac{2+1}{6}=\frac{\cancel{3}}{\cancel{6}}=\frac{1}{2}$$

Remember also not to exceed 2 units in either statement or answer, but let the children work through as many examples as possible within these limits, again writing only the answers :

(i) $\tfrac{1}{3}+\tfrac{1}{3}$; $\tfrac{1}{3}+\tfrac{2}{3}$; $\tfrac{2}{3}-\tfrac{1}{3}$; $1-\tfrac{1}{3}$; $1-\tfrac{2}{3}$.

(ii) $\tfrac{1}{3}+\tfrac{1}{6}$; $\tfrac{1}{6}+\tfrac{5}{6}$; $\tfrac{2}{3}+\tfrac{1}{6}$; $\tfrac{2}{3}-\tfrac{1}{6}$; $1-\tfrac{5}{6}$.

(iii) $\tfrac{2}{3}+\tfrac{2}{3}$; $\tfrac{2}{3}+\tfrac{5}{6}$; $1+\tfrac{2}{3}$; $1+\tfrac{5}{6}$; $1+1$; $1+\tfrac{1}{6}$.

(iv) $1\tfrac{5}{6}-\tfrac{1}{6}$; $1\tfrac{2}{3}-\tfrac{1}{3}$; $1\tfrac{2}{3}-\tfrac{2}{3}$; $2-1$; $2-\tfrac{2}{3}$; $2-\tfrac{1}{6}$.

(v) $\tfrac{1}{3}+\tfrac{1}{6}$; $1\tfrac{2}{3}+\tfrac{1}{6}$; $\tfrac{1}{3}-\tfrac{1}{6}$; $1\tfrac{1}{3}-\tfrac{1}{6}$; $1\tfrac{1}{3}+\tfrac{2}{3}$;
$1\tfrac{2}{3}-1\tfrac{1}{3}$; $1\tfrac{2}{3}-1\tfrac{1}{6}$; $2-\tfrac{1}{3}$; $2-1\tfrac{1}{3}$; $2-\tfrac{1}{6}$;
$2-1\tfrac{1}{6}$; $1\tfrac{1}{6}+\tfrac{5}{6}$; $1\tfrac{5}{6}-\tfrac{2}{3}$; $1\tfrac{1}{6}-\tfrac{2}{3}$; $\tfrac{2}{3}+\tfrac{5}{6}$.

It will be noticed that the terms $\tfrac{2}{6}$, $\tfrac{3}{6}$ and $\tfrac{4}{6}$ are avoided in the above questions. The teacher should avoid these likewise when making up his own examples. The children have already learnt that $\tfrac{2}{6}=\tfrac{1}{3}$, so this should always be written as $\tfrac{1}{3}$, and in the same way $\tfrac{4}{6}$ should always appear as $\tfrac{2}{3}$, both in question and answers. $\tfrac{3}{6}$ should be avoided in the setting of questions as there are plenty of alternative sums available, but as has already been stated above it should be accepted as correct whenever it appears in an answer at this stage.

Step 8. (About 1 lesson.) Relation of fractional parts to real life.

Immediately before being introduced to fractions the children were dealing with linear measure and this medium should be used, together with the 'yard' and 'foot' measures first used in

class 2, for the children to carry out simple practical measuring exercises working in groups and convert their findings to fractions as follows:

(*a*) 'How many feet did we say there are in one yard? (3) What part of one yard then is *one* foot?' (Elicit answer $\frac{1}{3}$ and then write down on the blackboard for all to see: 1 ft. is $\frac{1}{3}$ of 1 yd. and $\frac{1}{3}$ of 1 yd. is 1 ft.)

(*b*) Do the same for 2 ft. and then take both as a fraction of 2 yd. Then similarly discover the fractional relationships of 1 yd. 1 ft., and 1 yd. 2 ft. to 2 yd.

(*c*) The measuring of carefully selected items and comparing their lengths against the standard measure of either one yard or two yards.

Note carefully:

The teacher must have prepared this step carefully beforehand by measuring all of the items and choosing only those whose length will give as an answer one of the easy fractional parts already learnt.

NUMBER

LONG MULTIPLICATION

Multiplication by 2 digits is introduced. It must be emphasised that this should be introduced with great care as it is, for the children, an involved process. Children must be led to understand the process.

Step 1. In the first stage the fact is taught that multiplying a figure by 10 always produces an answer that is the same figure with a zero at the end.

Children will be asked to multiply as follows:

$$\begin{array}{r}3\\ \times 10\\ \hline\end{array}\qquad \begin{array}{r}6\\ \times 10\\ \hline\end{array}\qquad \begin{array}{r}5\\ \times 10\\ \hline\end{array}\qquad \text{etc.}$$

and if not already noted the teacher will get his class to note this feature of the same figure with a zero at the end. He will then proceed to:

$$\begin{array}{r}13\\ \times 10\\ \hline\end{array}\qquad \begin{array}{r}16\\ \times 10\\ \hline\end{array}\qquad \text{and so on to}\qquad \begin{array}{r}99\\ \times 10\\ \hline\end{array}$$

This must be thoroughly understood.

Step 2. The teacher then places the following sum on the blackboard:

$$\begin{array}{r}25\\ \times 3\\ \hline\end{array}$$

A child is required to do this sum. When the answer is found, the teacher will point out that the top line has been multiplied by 3 and write:

$$\begin{array}{r}25\\ \times 3\\ \hline 75\end{array} = 3 \times 25$$

Step 3. Then the teacher inserts the ten figure before the 3. He asks what figure he has put in. It is possible he will be told ' one '. He will elicit ' ten ' and ensure the class understands this as ' ten '. A child then multiplies the top line by 10, as drilled earlier, and the blackboard now reads :

$$\begin{array}{r@{}l} 25 & \\ \times 13 & \\ \hline 75 & = 3 \times 25 \\ 250 & = 10 \times 25 \end{array}$$

He gets from the class that the sum requires an answer that is 13×25 and that to get this it will be necessary to add the two answers, and the blackboard finally reads :

$$\begin{array}{r@{}l} 25 & \\ \times 13 & \\ \hline 75 & = 3 \times 25 \\ 250 & = 10 \times 25 \\ \hline 325 & = 13 \times 25 \end{array}$$

Note very carefully :

1. *Multiplication* by the unit figure *must* be done before multiplication by tens.

2. The zero in the unit column must always be written when multiplying by the tens.

The right-hand explanation will be written by the children in their subsequent exercises. The multiplier will not be greater than 19 until a few periods of applications have been done and the children are clearly perfectly at ease ; then introduce :

Step 4. Multiply by 2 digits beyond 19, with 2 digits only on the top line :

$$32 \times 22 ; \;\; 68 \times 24 ; \;\; 57 \times 27 ; \;\; 49 \times 32.$$

Note carefully :

The teacher is required to follow the principle as explained above of not going beyond 19 for many periods of exercises, and then doing it carefully. There will be no more than 2 figures in the top line.

CAPACITY

Aim

A practical introduction to capacity by means of simple experiments in class, and leading eventually to the table:

8 pints = 1 gallon

4 gallons = 1 debe

and sums covering the 4 rules.

Apparatus: A 4 gallon debe, one or more gallon oil tins, and several one pint mugs.

Introduction

With the class taking part, show that 8 pint mugs of water will fill a gallon tin. Then in the same way, 4 of these one gallon tins will fill a debe.

Let this be done a few times by children so that the table above has a real meaning.

ADDITION

Step 1. Addition of pints and prepare for gallons in answer. Do *not* have more than 7 pt. in any item.

		gall.	pt.		gall.	pt.		gall.	pt.
(i)			2			3			1
	+		3	+		4	+		2

		gall.	pt.		gall.	pt.		gall.	pt.
(ii)			4			4			6
	+		2	+		1	+		1

Step 2. Introduce carrying, and therefore, gallons in answer:

	gall.	pt.			gall.	pt.			gall.	pt.	
		6				6				7	
+		4		+		7		+		3	
	1	2	*Ans.*		1	5	*Ans.*		1	2	*Ans.*
	1	8)10			1	8)13			1	8)10	
		1 r 2				1 r 5				1 r 2	

Make 10 sums of this type.

Step 3. Introduce 3, then 4, items :

gall.	pt.		gall.	pt.		gall.	pt.	
	6			7			7	
	4			5			5	
+	2		+	6		+	4	
1	4	*Ans.*	2	2	*Ans.*	2	0	*Ans.*
1	8)12		2	8)18		2	8)16	
	1 r 4			2 r 2			2 r 0	

Make 10 sums of this type.

(*b*)

gall.	pt.		gall.	pt.		gall.	pt.	
	4			5			4	
	6			8			3	
	7			2			1	
+	3		+	6		+	7	
2	4	*Ans.*	2	5	*Ans.*	1	7	*Ans.*
2	8)20		2	8)21		1	8)15	
	2 r 4			2 r 5			1 r 7	

Make 10 sums of this type.

Step 4. Addition of unit gallons and pints (with gallons in top line only at first, then in bottom line only, and finally appearing in both lines. Do *not* have more than 3 gall. in any item. *No* carrying.

	gall.	pt.	gall.	pt.	gall.	pt.
(*a*)	1	2	1	3	2	5
	+	3	+	4	+	1
	gall.	pt.	gall.	pt.	gall.	pt.
(*b*)		3		4		2
	+ 1	2	+ 1	1	+ 2	5
	gall.	pt.	gall.	pt.	gall.	pt.
(*c*)	1	2	2	2	1	5
	+ 1	3	+ 1	4	+ 2	1

Make 5 sums of each

Step 5. Proceed by same method as in Step 4, to 2, 3 and 4 items, *with* carrying.

(*a*)

	gall.	pt.	
	1	4	
+		6	
	2	2	*Ans.*
	1	8)10	
	1	1 r 2	
	2		

	gall.	pt.	
		4	
+	1	6	
	2	2	*Ans.*
	1	8)10	
	1	1 r 2	
	2		

	gall.	pt.	
	1	6	
+	1	5	
	3	3	*Ans.*
	1	8)11	
	2	1 r 3	
	3		

Make 20 sums for the class to do.

(*b*)

	gall.	pt.	
	1	0	
		6	
+		4	
	2	2	*Ans.*
	1	8)10	
	1	1 r 2	
	2		

	gall.	pt.	
	1	2	
		7	
+	1	1	
	3	2	*Ans.*
	1	8)10	
	2	1 r 2	
	3		

	gall.	pt.	
	1	1	
	1	2	
+	1	4	
	3	7	*Ans.*

Make 20 sums for the class to do.

(*c*)

	gall.	pt.	
		6	
	1	1	
		3	
+		2	
	2	4	*Ans.*
	1	8)12	
	1	1 r 4	
	2		

	gall.	pt.	
	1	0	
		7	
	1	1	
+		3	
	3	3	*Ans.*
	1	8)11	
	2	1 r 3	
	3		

	gall.	pt.	
		6	
		4	
		7	
+	1	2	
	3	3	*Ans.*
	2	8)19	
	1	2 r 3	
	3		

Make 20 sums for the class to do.

Step 6. Introduce debe in answer only, with 2, 3 then 4 items.

(*a*) No carrying from pt.

	de.	gall.	pt.	
		2	4	
+		3	1	
	1	1	5	*Ans.*
	1	4)5		
		1 r 1		

(*b*) Carrying from both quantities.

	de.	gall.	pt.	
		2	4	
+		3	7	
	1	2	3	*Ans.*
	1	1	8)11	
		5	1 r 3	
		4)6		
		1 r 2		

(*c*) 3 items, no carrying from pt.

	de.	gall.	pt.	
		2	1	
		4	2	
+		3	2	
	2	1	5	*Ans.*
	2	4)9		
		2 r 1		

(*d*) Carring from both quantities.

	de.	gall.	pt.	
		3	2	
		5	7	
+		1	4	
	2	2	5	*Ans.*
	2	1	8)13	
		9	1 r 5	
		4)10		
		2 r 2		

(*e*) 4 items, no carrying from pt.

	de.	gall.	pt.	
		2	1	
		1	3	
		2	0	
+		1	2	
	1	2	6	*Ans.*
	1	4)6		
		1 r 2		

(*f*) Carrying from both quantities.

	de.	gall.	pt.	
		2	4	
		3	6	
		4	7	
+		6	2	
	4	1	3	*Ans.*
	4	2	8)19	
		15	2 r 3	
		4)17		
		4 r 1		

Make at least 6 sums of each kind.

Step 7. Addition of de. gall. and pt. first by two items, then by 3 and then by 4 items:

	de.	gall.	pt.	
(*a*)	1	2	3	
	+1	3	1	
	3	1	4	*Ans.*
	1	4)5		
	2	1 r 1		
	3			

	de.	gall.	pt.	
(*b*)	2	2	6	
	+3	3	4	
	6	2	2	*Ans.*
	1	1	8)10	
	5	5	1 r 2	
	6	4)6		
		1 r 2		

	de.	gall.	pt.	
(*c*)	1	2	3	
	1	3	1	
	+1	0	2	
	4	1	6	*Ans.*
	1	4)5		
	3	1 r 1		
	4			

	de.	gall.	pt.	
(*d*)	1	2	3	
		3	7	
	+2	0	1	
	4	2	3	*Ans.*
	1	1	8)11	
	3	5	1 r 3	
	4	4)6		
		1 r 2		

	de.	gall.	pt.	
(*e*)	1	1	1	
	2	3	2	
	3	1	3	
	+2	2	0	
	9	3	6	*Ans.*
	1	4)7		
	8	1 r 3		
	9			

	de.	gall.	pt.	
(*f*)	1	2	3	
	4	3	7	
		3	4	
	+1	0	5	
	8	2	3	*Ans.*
	2	2	8)19	
	6	8	2 r 3	
	8	4)10		
		2 r 2		

Make 10 sums of each kind.

SUBTRACTION

Step 1. Subtraction of pints, firstly from pints only, then from gallons and pints, with *no* carrying.

(*a*)

pt.	pt.	pt.	pt.
7	6	5	7
−3	−1	−2	−6

(*b*)

	gall.	pt.		gall.	pt.		gall.	pt.
	1	6		1	5		3	7
−		2	−		4	−		4

Make 10 sums of this type.

Step 2. Introduction of carrying figure: at first it will be advisable to cross out and insert as in example below:

	gall.	pt.			gall.	pt.			gall.	pt.	
	1	12			2	11				12	
	~~2~~	~~4~~			~~3~~	~~3~~			~~1~~	~~4~~	
−		6		−		7		−		7	
	1	6	*Ans.*		2	4	*Ans.*			5	*Ans.*

Make 10 sums of this type.

Step 3. Introduction of gallons in bottom line, first without, then with carrying.

(*a*)

	gall.	pt.
	1	4
−	1	2

(*b*)

	gall.	pt.		gall.	pt.
	3	4		3	0
−	1	6	−	2	1

Make 5 sums of each (*a*) and (*b*).

Step 4. Introduction of debe with gallons and pints; (*a*) firstly without carrying, (*b*) then with carrying from gallons only, then (*c*) from debes, (*d*) and finally from both quantities.

(*a*)

de.	gall.	pt.	de.	gall.	pt.	de.	gall.	pt.
2	3	6	2	3	6	2	1	6
−1	2	3	−1	3	4	−	1	4

Make 5 sums of this type.

(*b*)

	de.	gall.	pt.	
	2	3	4	
	−1	1	7	
	1	1	5	*Ans.*

de.	gall.	pt.	
3	3	3	
−1	0	6	
2	2	5	*Ans.*

de.	gall.	pt.
4	2	1
−4	1	2

Make 10 sums of this type.

(*c*)

de.	gall.	pt.
3	1	6
−1	2	4

(*d*)

de.	gall.	pt.
3	1	4
−1	2	6

Make 20 sums of this type.

Step 5. Carrying from a zero in the gall. column. This needs to be well taught with emphasis on inserting the new debe figure after carrying and inserting the new 4 gall. figure in place of the zero; then crossing through this figure to insert a 3, carrying a gallon to the pt. column. The children must be allowed to do the insertions as shown in the example below. Then give at least 10 sums of this kind.

de.	gall.	pt.
2	$\cancel{4}$ 3	
$\cancel{3}$	$\cancel{0}$	4
−2	3	6

MULTIPLICATION

Step 1. Short multiplication only is to be done. Simple multiplication of pints leading quickly to answers in the gallons column.

(*a*)

pt.	
3	
×2	
6	*Ans.*

(*b*)

gall.	pt.	
	3	
	×4	
1	4	*Ans.*
1	8)12	
	1 r 4	

gall.	pt.	
	7	
	×4	
3	4	*Ans.*
3	8)28	
	3 r 4	

Step 2.

(*a*) Multiplication of gallons and pints.

gall.	pt.	gall.	pt.	gall.	pt.
1	2	1	4	1	2
	×2		×2		×3
2	4	3	0	3	6
		1	8)8		
			1 r 0		

Make 10 sums of this kind.

(*b*) Multiplication of gall. and pt. with debes in answers.

de.	gall.	pt.		de.	gall.	pt.		de.	gall.	pt.	
	2	1			1	1			2	3	
		×3				×5				×2	
1	2	3	*Ans.*	1	1	5	*Ans.*	1	0	6	*Ans.*
1	4)6			1	4)5			1	4)4		
	1 r 2				1 r 1				1 r 0		

Make 5 sums of this kind.

(*c*) Multiplication of pt. with de. gall. pt. in answer.

de.	gall.	pt.		de.	gall.	pt.		de.	gall.	pt.	
		3				5				7	
		×11				×8				×11	
1	0	1	*Ans.*	1	1	0	*Ans.*	2	1	5	*Ans.*
1	4)4	8)33		1	4)5	8)40		2	4)9	8)77	
	1 r 0	4 r 1			1 r 1	5 r 0			2 r 1	9 r 5	

Make 10 sums of this kind.

Step 3. Introduction of debes in the top line; (*a*) first with no carrying, (*b*) then single carrying from one quantity, and then (*c*) carrying two quantities.

(*a*)

de.	gall.	pt.		de.	gall.	pt.
4	1	2		3	0	3
		×3				×2

Make 5 sums of this kind.

(*b*)

de.	gall.	pt.	
2	0	4	
		×7	
14	3	4	*Ans.*
	3	8)28	
		3 r 4	

de.	gall.	pt.	
2	3	1	
		×5	
13	3	5	*Ans.*
3	4)15		
10	3 r 3		
13			

de.	gall.	pt.	
2	1	7	
		×2	
4	3	6	*Ans.*
	1	8)14	
	2	1 r 6	
	3		

de.	gall.	pt.	
4	1	0	
		×11	
46	3	0	*Ans.*
2	4)11		
44	2 r 3		
46			

Make 20 sums of this kind.

(*c*)

de.	gall.	pt.	
3	2	3	
		×9	
32	1	3	*Ans.*
5	3	8)27	
27	18	3 r 3	
32	4)21		
	5 r 1		

de.	gall.	pt.	
4	3	7	
		×11	
54	2	5	*Ans.*
10	9	8)77	
44	33	9 r 5	
54	4)42		
	10 r 2		

Make 20 sums of this kind.

DIVISION

Step 1. Short division only is to be done. (*a*) Simple short division of gallons and pints (1 period only) without carrying figures.

(*a*)

gall.	pt.	
3)3	6	
1	2	*Ans.*

gall.	pt.	
2)2	6	
1	3	*Ans.*

gall.	pt.	
3)3	3	
1	1	*Ans.*

(*b*) With carrying figures.

$$\begin{array}{rrr} & \text{gall.} & \text{pt.} \\ & & 14 \\ 2) & 3 & \cancel{6} \\ \hline & 1 & 7 \\ \hline \end{array}\ \textit{Ans.} \qquad \begin{array}{rrr} & \text{gall.} & \text{pt.} \\ & & 20 \\ 4) & 6 & \cancel{4} \\ \hline & 1 & 5 \\ \hline \end{array}\ \textit{Ans.} \qquad \begin{array}{rrr} & \text{gall.} & \text{pt.} \\ & & 24 \\ 6) & 3 & \cancel{0} \\ \hline & & 4 \\ \hline \end{array}\ \textit{Ans.}$$

Make 10 sums of this kind.

Step 2. (*a*) Simple division of de. gall. pt., first of all with no carrying, then (*b*) with carrying to pints only.

$$(a)\quad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ 3) & 6 & 3 & 6 \\ \hline & 2 & 1 & 2 \\ \hline \end{array}\ \textit{Ans.} \qquad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ 4) & 8 & 0 & 4 \\ \hline & 2 & 0 & 1 \\ \hline \end{array}\ \textit{Ans.}$$

Make 5 sums of this kind.

$$(b)\quad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ & & & 15 \\ 3) & 6 & 4 & \cancel{7} \\ \hline & 2 & 1 & 5 \\ \hline \end{array}\ \textit{Ans.} \qquad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ & & & 10 \\ 5) & 5 & 1 & \cancel{2} \\ \hline & 1 & 0 & 2 \\ \hline \end{array}\ \textit{Ans.}$$

Make 10 sums of this kind.

Step 3. (*a*) Introduction of carrying to gallons, and then (*b*) proceed to carrying in 2 quantities.

$$(a)\quad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ & & 6 & \\ 3) & 7 & \cancel{2} & 6 \\ \hline & 2 & 2 & 2 \\ \hline \end{array}\ \textit{Ans.} \qquad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ & & 24 & \\ 8) & 14 & \cancel{0} & 0 \\ \hline & 1 & 3 & 0 \\ \hline \end{array}\ \textit{Ans.}$$

Make 10 sums of this kind.

$$(b)\quad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ & & 5 & 18 \\ 3) & 7 & \cancel{1} & \cancel{2} \\ \hline & 2 & 1 & 6 \\ \hline \end{array}\ \textit{Ans.} \qquad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ & & 23 & 11 \\ 11) & 27 & \cancel{3} & \cancel{3} \\ \hline & 2 & 2 & 1 \\ \hline \end{array}\ \textit{Ans.}$$

$$\begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ & & 14 & 16 \\ 4) & 7 & \cancel{2} & \cancel{0} \\ \hline & 1 & 3 & 4 \\ \hline \end{array}\ \textit{Ans.} \qquad \begin{array}{rrrr} & \text{de.} & \text{gall.} & \text{pt.} \\ & & 24 & 54 \\ 9) & 24 & \cancel{0} & \cancel{6} \\ \hline & 2 & 2 & 6 \\ \hline \end{array}\ \textit{Ans.}$$

Make 10 sums of this kind.

Step 4. Introduction of remainders. It is essential that remainders in excess of 7 pts. should not appear and this should be borne in mind by teachers when they make up their own examples.

	de.	gall.	pt.	
		7	31	
4)	5	$\cancel{3}$	$\cancel{7}$	
	1	1	7 r 3 pt.	*Ans.*

	de.	gall.	pt.	
			21	
9)	9	2	$\cancel{5}$	
	1	0	2 r 3 pt.	*Ans.*

	de.	gall.	pt.	
		39	48	
11)	9	$\cancel{3}$	$\cancel{0}$	
		3	4 r 4 pt.	*Ans.*

	de.	gall.	pt.	
		21		
7)	19	$\cancel{1}$	5	
	2	3	0 r 5 pt.	*Ans.*

Make 10 sums of this kind.

WEIGHT

Introduction

The preliminary work on weight has been left to Class 3 because most of the mechanical processes connected with this topic require a knowledge of long division, which is not begun until Class 4. It will be noticed that two of the weights common in England but little used, if at all, in East Africa, have been left out of this manual. The stone (14 lb.) is not used at all, and the quarter (28 lb.) only for certain kinds of crop and the weight of some vehicles. The weights used in this handbook are therefore the ounce (oz.) the pound (lb.) the hundred-weight (cwt.) and the ton.

Practical Work

Every school should be provided with a set of scales. These are only useful for teaching the oz. and lb. weights, but much work can be done with them. The best type of scale for use is the type with two pans and a set of weights from 1 oz. to 7 lb. A specimen of this can be seen in most general shops in East Africa. Other small types of scale are rather risky things to have in schools, because they are easily damaged: but the teacher should know about them himself and should talk about them

with his pupils—for instance the letter-scale at the Post Office, the spring balance and the housewife's scale for measuring food. Any of these which can be borrowed for a period should be obtained and a dramatised lesson planned to fit the borrowed apparatus. Perhaps there is a market in the nearby village or town where the butcher has a beam-scale for weighing quarters of meat. Find out how it works, try to take the class to see it: or find out who *has* seen it and get him to explain to the rest of the class. There may be a large scale (usually called a 'weigh-bridge' in English) which is used to weigh motor lorries. Take the class to have a look at it and try to persuade the inspector in charge to weigh a car or lorry for them.

Whether scales are available or not, have a number of bags of sand, *accurately weighed*, of 1, 2, 4 and 8 oz., and 1 lb. weights. There should be sufficient of the smaller bags to show in various ways the fact that in 1 lb. there are 16 oz., e.g. at least 2 bags of 8 oz. each, 4 bags of 4 oz. each, etc. This practical work is based on the assumption that there are, in fact, both scales and sand-bags in the school, and that the teacher takes trouble to find other things for the class to weigh, such as boxes of chalk, bean bags, bundles of books, earth, stones. The general form of explanatory talk can be made by the teacher himself by adapting the detailed explanations given for the teaching of Linear Measure in Class 2.

Step 1. Introduce the pound weight.

Show the children a 1 lb. weight and tell them what it is. Weigh one lb. against another. Then let groups of pupils weigh other objects—which you have previously tested for accuracy—which are exactly 1 lb. in weight, and let them record the results on paper or in their arithmetic books. Explain how a scale works when weighing the first lb. yourself.

Step 2. 1 lb. = 16 oz.

Have handy a few objects which weigh more than 1 lb. but less than 2 lbs. Demonstrate—and allow the class to prove for themselves—that these objects are heavier than 1 lb. Then show how the 'over-weight' can be measured by using oz. weights. Show that these oz. weights are a unit of measure by weighing one against another, and weighing an oz. sandbag against a weight.

Tell the class what this new small weight is called. Then allow them to see, handle, and weigh the 2 oz., 4 oz. and 8 oz. weights. Finally weigh the objects using the 1 lb. weight and the correct number of oz. weights and have the children record the results as before. Weigh and learn '1 lb. = 16 oz.'

Step 3. Chief fractions of 1 lb.

Now weigh—and allow the children to weigh—objects whose weight is less than 1 lb., and record the results as before. During this step weigh and learn '$\frac{1}{2}$ lb. = 8 oz., $\frac{1}{4}$ lb. = 4 oz., $\frac{3}{4}$ lb. = 12 oz.', and have some objects which are exactly these fractional weights.

Step 4. Heavier weights in lb. and oz.

At the beginning of this step the heavier weights (2–7 lb.) should be introduced to the class in the same way as the smaller weights. Allow the class to weigh and record the weight of any object they wish, e.g. a football, a pumpkin, a pawpaw, the globe used in geography.

Practice without scales.

Where there are no scales in the school, the children should at least have the opportunity to feel, handle and judge comparative weights by using the sandbags referred to above. They should also be trained to balance objects against the weights and make approximate judgments of the weight of the objects. A straight short plank of wood and a short length of *straight* tree trunk might be rigged up into a rough kind of balance, and the working of a scale demonstrated by means of this apparatus. Every effort, however, should be made to equip each school with weight apparatus, including scales.

Mechanical Work

Step 1. Addition without carrying.

'Without carrying' means, of course, without carrying from oz. to lb. The best way to give this type of work at this stage is to select very carefully the weights of actual objects which the class can first weigh for themselves, and then let them add two weights together.

(*a*) *Lb. only.* An example of procedure is as follows :

Take two pineapples, etc., weighing exactly 2 lb. and 1 lb. respectively. Let one pupil weigh the first pineapple and the teacher write the weight on the blackboard. Repeat with the second pineapple and ask how much the pineapples weigh together. The class will most likely give the answer straight away. Do the sum mechanically on the blackboard :

lb.	oz.	
2	0	
+1	0	
3	0	*Ans.*

Only the mechanical sum is to appear on the blackboard. *No words.*

Lastly check the answer by weighing both pineapples together.

(*b*) *Oz only.*

The same form of approach leading to sums as below :

lb.	oz.	
	11	
+	4	
	15	*Ans.*

Spend at least one period on these two types of sum before going on to the third type.

(*c*) *Lb. and oz.*

The best way again is by adding the weights of real objects :

lb.	oz.	
3	9	
+4	6	
7	15	*Ans.*

Revise frequently the rules for setting down—which are the same as for linear measure sums—and the table item ‘ 16 oz = 1 lb.’.

Step 2. Subtraction without carrying.

Use the same approach as for addition and the same three sections. The first two can be done in one period in most cases :

(*a*) *lb. only.*

$$\begin{array}{rr} \text{lb.} & \text{oz.} \\ 9 & 0 \\ -5 & 0 \\ \hline 4 & 0 \\ \hline \end{array}\ \textit{Ans.}$$

(*b*) *oz. only from lb. oz.*

$$\begin{array}{rr} \text{lb.} & \text{oz.} \\ 2 & 15 \\ - & 8 \\ \hline 2 & 7 \\ \hline \end{array}\ \textit{Ans.}$$

(*c*) *lb. and oz.*

$$\begin{array}{rr} \text{lb.} & \text{oz.} \\ 7 & 14 \\ -3 & 9 \\ \hline 4 & 5 \\ \hline \end{array}\ \textit{Ans.}$$

Step 3. Addition and subtraction with fractional answer.

The class have already learned (Practical Work) the equivalents of $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ lb. Point out to them that it is the usual thing to talk of $2\frac{1}{4}$ lb. rather than 2 lb. 4 oz., etc. Then demonstrate an addition and a subtraction sum, with the result expressed again in fractional form :

$$\begin{array}{rr} \text{lb.} & \text{oz.} \\ 6 & 3 \\ +\ 5 & 9 \\ \hline 11 & 12 \\ \hline \end{array} = 11\tfrac{3}{4}\text{ lb. } \textit{Ans.} \qquad \begin{array}{rr} \text{lb.} & \text{oz.} \\ 12 & 15 \\ -\ 9 & 7 \\ \hline 3 & 8 \\ \hline \end{array} = 3\tfrac{1}{2}\text{ lb. } \textit{Ans.}$$

Give at least twenty examples in this step—ten of addition and ten of subtraction. The practical approach may be dispensed with. See that there is no crushing of figures together in the answer line.

Where the school has no scales and the fullest practical approach cannot be carried out, the teacher should do his best to make up for this by using dummy objects clearly marked with an appropriate weight.

BILLS

A more advanced stage should now be developed on the style learnt in Class 2. Articles should be so priced that the class can work each item *mentally*, e.g. 4 lb. of salt at 50 ct. a lb. is preferable to 4 lb. of salt at 58 ct. a lb. if it is known that some of the class may fail at this. The ½ and ¼ fractions should be introduced towards the end after this has been thoroughly understood in earlier instruction in the mental part of the lessons. Reach a maximum of 4 items.

For example :

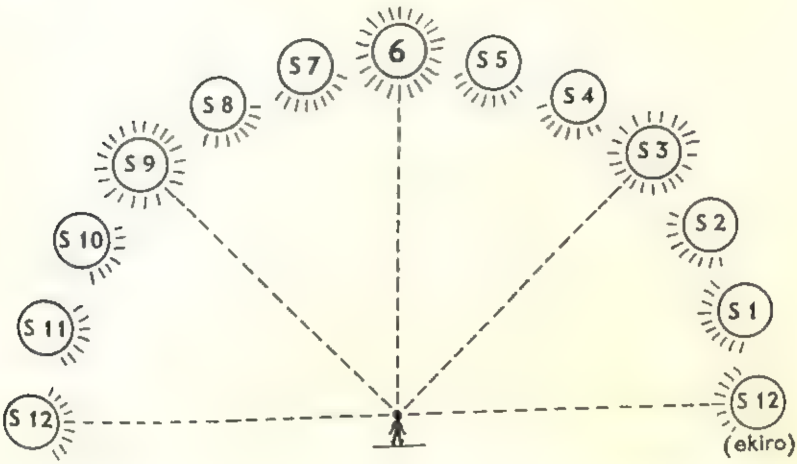
½ lb. of sugar at 68 ct. a lb.
4 lb. of flour at 60 ct. a lb.
2 de. of paraffin at 12.50 each.
¼ lb. of meat at 2.00 a lb.

TIME

In this class the pupils are to be taught to express and understand ' sun ' time in their own vernacular languages.

Step 1. On the day before the first period is to be given the whole class should be taken outside for a minute or so at approximately hourly intervals to note the progress of the sun across the sky with the passing hours. (*Note:* It is particularly important that this be done at the end of morning school, when the sun is at its highest, and also right at the beginning and end of the day.)

Step 2. The teacher should make a chart on the following lines :



From this chart the children can learn the connections between the time of day, the position of the sun and the numerical expression of the hour.

Further, the teacher should himself watch the daily progress of the sun and attempt to pinpoint landmarks indicating one or more half-hours, and the idea of an intermediate half-hour can then be introduced.

As the telling of time in the vernacular is traditionally a very general and approximate affair it is not necessary to attempt to introduce any subdivisions smaller than the half-hour.

FRACTIONS

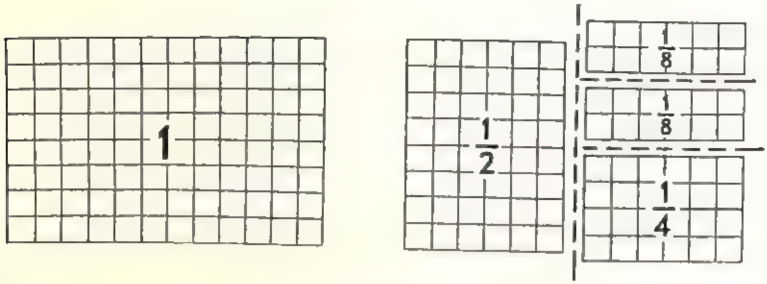
Step 1. First of all a brief but thorough revision of the work done earlier in the year in steps 1–8 should be carried out, culminating in the working by the children of 10 or more questions at the Step 5 level. This should not take more than one lesson.

Step 2. Introduction of eighths.

The children should now be familiar with the actions and thoughts involved and if they are provided with the usual pieces

of paper and the teacher asks suitable questions they should very soon proceed through the following stages :

(*a*) Division and numbering of the second piece of paper :



(*b*) Appreciation of fact that $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ and thus that while there are 4 quarters in one whole unit, there are 8 eighths.

(*c*) Simple fractional additions and subtractions with eighths with the children using their paper fractions as a check and going directly to the answer, as in the following :

(i) $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$. $\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$.

Then proceed through the following stages, the children writing *only* the answers :

(ii) $\frac{1}{8}+\frac{1}{4}$; $\frac{3}{8}+\frac{1}{4}$; $\frac{3}{8}+\frac{1}{8}$; $\frac{3}{8}-\frac{1}{4}$;
$\frac{1}{2}-\frac{1}{4}$; $\frac{1}{2}-\frac{1}{8}$; $\frac{3}{8}-\frac{1}{8}$.

(iii) $\frac{1}{2}+\frac{1}{4}$; $\frac{1}{2}+\frac{1}{8}$; $\frac{1}{2}+\frac{3}{8}$; $\frac{1}{2}+\frac{1}{2}$; $\frac{3}{4}+\frac{1}{8}$; $1-\frac{1}{2}$; $1-\frac{1}{4}$;
$1-\frac{1}{8}$; $\frac{3}{4}-\frac{1}{4}$; $\frac{3}{4}-\frac{1}{8}$.

(iv) $\frac{5}{8}+\frac{1}{4}$; $\frac{5}{8}+\frac{1}{8}$; $\frac{5}{8}+\frac{3}{8}$; $1-\frac{1}{4}$; $1-\frac{3}{8}$;
$\frac{7}{8}+\frac{1}{8}$; $\frac{7}{8}-\frac{3}{8}$; $\frac{7}{8}-\frac{1}{8}$; $\frac{7}{8}-\frac{3}{8}$; $\frac{7}{8}-\frac{1}{4}$.

(v) $\frac{1}{8}+\frac{5}{8}$; $\frac{3}{4}-\frac{5}{8}$; $\frac{7}{8}+\frac{1}{8}$; $1-\frac{5}{8}$; $\frac{1}{4}+\frac{5}{8}$;
$\frac{3}{8}+\frac{3}{8}$; $\frac{7}{8}-\frac{5}{8}$; $\frac{1}{8}+\frac{3}{4}$; $\frac{3}{4}-\frac{3}{8}$, etc.

Step 3. The application of halves, quarters, and eighths to reality, by reference to weight and capacity. This should be kept to a very simple level and follow the same outline as that used in step 8 of term 2 for the relating of fractional parts to linear measure.

(*a*) Weight. This can be kept to halves and quarters only and

is really only a revision and expansion of the fractional work already done in dealing with weight earlier in the year.

(i) First ensure that the children remember that:

$$\begin{array}{ll} & 8 \text{ oz.} = \tfrac{1}{2} \text{ lb.} \\ \text{and} & 4 \text{ oz.} = \tfrac{1}{4} \text{ lb.} \end{array}$$

then (ii)

$$\left.\begin{array}{r} \tfrac{1}{4} \text{ lb.} + \tfrac{1}{2} \text{ lb.} = \\ 4 \text{ oz.} + \tfrac{1}{2} \text{ lb.} = \\ \tfrac{1}{4} \text{ lb.} + 8 \text{ oz.} = \\ 12 \text{ oz.} - \tfrac{1}{4} \text{ lb.} = \\ 1 \text{ lb.} - 8 \text{ oz.} = \end{array}\right\}$$

Answer always to be given as a fraction of a pound, and *not* in ounces. Answers *only* to be written.

(*b*) Capacity follows much the same lines, but this time deal mainly with eighths, having first revised 8 pints = 1 gallon and from that enable the children to discover that:

$$1 \text{ pint} = \tfrac{1}{8} \text{ gallon}$$

Then proceed to this type of question:

$$\left.\begin{array}{l} 4 \text{ pt.} + \tfrac{1}{8} \text{ gall.} = \\ 1 \text{ gall.} - 4 \text{ pt.} = \\ \tfrac{1}{4} \text{ gall.} + 3 \text{ pt.} = \\ 5 \text{ pt.} - \tfrac{1}{4} \text{ gall.} = \\ 7 \text{ pt.} + \tfrac{1}{8} \text{ gall.} = \\ \tfrac{1}{4} \text{ gall.} + 5 \text{ pt.} = \\ 6 \text{ pt.} - \tfrac{1}{2} \text{ gall.} = \end{array}\right\}$$

and so on

Again all answers to be given in fractions of gallons and *not* in pints. Answers *only* to be written.

Step 4. The composition and meaning of fractional parts.

The explanations here must be kept very simple and clear and must be related in every instance to the work done in Step 3 above (and also Step 8 of Term 2) and should be accompanied by practical work done by the children working in small groups.

(*a*) *The meaning of ½ and ¼.*

(i) After introductory statements to refresh the children's memory the teacher asks 'How many ounces did we say there are in ½ lb.?' (8) 'How many oz. in 1 lb.' (16). Then proceed, getting answers from the children

wherever possible to $16 \div 2 = 8$ and leading eventually to the writing on the blackboard of the statements :

$$1 \text{ lb.} \div 2 = \tfrac{1}{2} \text{ lb.*}$$

and $$\tfrac{1}{2} \text{ lb.} = 1 \text{ lb.} \div 2$$

Then do the same for 4 oz. ($\frac{1}{4}$ lb.).

So far this is merely a revision in another medium of the work done by dividing pieces of paper.

(ii) We have

$$1 \text{ lb.} \div 4 = \tfrac{1}{4} \text{ lb.*}$$

Now suppose we have

$$3 \text{ lb.} \div 4?$$

(Children working in their groups should first amass 2 oz., 4 oz. and 8 oz. weights or bags to total 3 lb., and then divide the pile into 4 equal groups and check the weight of one group.)

How much? (12 oz.) How much is 12 oz. as a part of 1 lb.? ($\frac{3}{4}$)

Thus the statement is completed, becoming :

$$3 \text{ lb.} \div 4 = \tfrac{3}{4} \text{ lb.*}$$

(*b*) *The meaning of $\frac{1}{3}$ and $\frac{2}{3}$.*

(i) The weight apparatus is now replaced by yard and foot measures, the children again working in small groups.

(ii) By similar steps, with the children providing each of the answers, proceed to :

$$1 \text{ yd.} \div 3 = \tfrac{1}{3} \text{ yd.}$$

and $$2 \text{ yd.} \div 3 = \tfrac{2}{3} \text{ yd.}$$

(*c*) *The meaning of $\frac{1}{8}$, etc.*

(i) By this time the teacher should be able to do a single demonstration himself with a debe, a gallon can and a large number of small containers. The children still take part though, by counting and giving the answers.

* *Do not rub these three statements off the blackboard.*

(ii) Proceed to the answers:

1 gall. $\div 8 = \frac{1}{8}$ gall.

3 gall. $\div 8 = \frac{3}{8}$ gall.

(5 gall. $\div 8 = \frac{5}{8}$ gall.—if enough equipment is available).

(*d*) (i) The blackboard now reads:

1 lb. $\div 2 = \frac{1}{2}$ lb. and $\frac{1}{2}$ lb. $= 1$ lb. $\div 2$

1 lb. $\div 4 = \frac{1}{4}$ lb. and $\frac{1}{4}$ lb. $= 1$ lb. $\div 4$

3 lb. $\div 4 = \frac{3}{4}$ lb. and $\frac{3}{4}$ lb. $= 3$ lb. $\div 4$

1 yd. $\div 3 = \frac{1}{3}$ yd.

2 yd. $\div 3 = \frac{2}{3}$ yd.

1 gall. $\div 8 = \frac{1}{8}$ gall.

3 gall. $\div 8 = \frac{3}{8}$ gall.

5 gall. $\div 8 = \frac{5}{8}$ gall.

and if they have been recording the results of their experiments in their books, each child's arithmetic book will have this same record in it.

(ii) From this point it should not be long, if a suitable lead is given by the teacher, before the brighter children realise that the fractional part is always made up of an amount to be divided on the top and the number of parts into which it is to be divided underneath (i.e. denominator = divisor, and numerator = dividend, but do not use these complicated terms at present: rather concentrate upon making sure that the relationship has been fully understood).

If the class as a whole finds this idea difficult to understand revert to working in groups each with several pieces of paper and work through these same examples again, together with several others, in pure number.

(*e*) Only when stage (*d*) above has been fully mastered do the children proceed to do the following examples, the teacher doing the first *two* as demonstrations in each case:

$\frac{1}{8} = 1 \div 8$

$\frac{5}{8} = 5 \div 8$

(i) $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{7}{8}$.

(ii) $3 \div 8 = \frac{3}{8}$;
$1 \div 6 = \frac{1}{6}$;
$5 \div 8$; $3 \div 4$; $1 \div 2$; $2 \div 3$;
$1 \div 3$; $1 \div 6$; $7 \div 8$; $1 \div 1$.

(iii) Then mixed examples:
$3 \div 8$; $\frac{1}{6}$; $\frac{1}{4}$; $5 \div 6$; $\frac{1}{3}$; $3 \div 4$;
$\frac{7}{8}$; $\frac{1}{4}$; $1 \div 3$; 1; $\frac{2}{3}$; $5 \div 8$; $\frac{1}{8}$; $1 \div 2$.

NUMBER

ADDITION

Aim

The addition of five items up to a maximum answer of 99,999. Each step here will need only 1 period.

Step 1. Introduce Th Carrying up to H. Th in one line only.

1,649	1,384	2,486	
+ 281	+ 219	+ 327	etc.

Step 2. Carrying of H to Th.

1,649	1,384	2,486	3,582	
+ 438	+ 918	+ 917	+ 194	etc.

Step 3. Th in both lines.

1,649	1,384	2,486	2,582		4,186
+1,438	+1,918	+1,917	+2,370	to	+4,907

Step 4. Introduce 10 Th in one line, carrying up to Th only.

15,821	14,731	
+ 1,467	+ 3,849	etc.

Step 5. Carrying to 10 Th.

15,821	14,731	16,049	
+ 5,321	+ 7,021	+ 6,628	etc.

Step 6. 10 Th in both lines.

15,821	14,731	23,721		63,429
+15,369	+17,209	+18,921	etc. to	+29,064

Now increase to three items and give exercises on addition of three items; when well done, proceed to four, and then to five items.

The teacher should introduce a few problems on addition of number, no number to have more than five digits.

SUBTRACTION

Aim

Up to a maximum figure of 99,999 on top line.

Step 1. Introduce carrying from Th to H, with Th only in top line.

$$\begin{array}{r} 1{,}063 \\ -\ \ 431 \\ \hline \end{array} \qquad \begin{array}{r} 2{,}089 \\ -\ \ 645 \\ \hline \end{array} \qquad \begin{array}{r} 3{,}077 \\ -\ \ 864 \\ \hline \end{array}$$

Step 2. Carrying from any or all places.

$$\begin{array}{r} 1{,}363 \\ -\ \ 431 \\ \hline \end{array} \qquad \begin{array}{r} 2{,}639 \\ -\ \ 745 \\ \hline \end{array} \qquad \begin{array}{r} 3{,}577 \\ -\ \ 869 \\ \hline \end{array}$$

Step 3. Th in both lines with carrying from any or all places.

$$\begin{array}{r} 5{,}363 \\ -2{,}589 \\ \hline \end{array} \qquad \begin{array}{r} 6{,}389 \\ -2{,}891 \\ \hline \end{array} \qquad \begin{array}{r} 9{,}485 \\ -1{,}973 \\ \hline \end{array}$$

Step 4. Carrying *across* a zero.

$$\begin{array}{r} 4{,}205 \\ -1{,}329 \\ \hline \end{array} \qquad \begin{array}{r} 4{,}025 \\ -1{,}343 \\ \hline \end{array}$$

Step 5. Carrying from Tens of Th to Th with Tens of Th in top line.

$$\begin{array}{r} 10{,}632 \\ -\ \ 5{,}821 \\ \hline \end{array}$$

Step 6. Carrying from any or all places.

$$\begin{array}{r} 14{,}248 \\ -\ \ 4{,}369 \\ \hline \end{array}$$

Step 7. Tens of Th in both lines with carrying from any or all places.

$$\begin{array}{r} 15{,}248 \\ -13{,}329 \\ \hline \end{array}$$

MULTIPLICATION

Aim

To increase the top line to 3, then 4 digits in one figure multiplication.

Step 1.

106	234	392	408	
$\times 7$	$\times 6$	$\times 8$	$\times 8$	etc.

Step 2.

1,132	2,496	
$\times 7$	$\times 8$	etc.

Step 3.

2,341	5,032
$\times 6$	$\times 8$

LONG MULTIPLICATION

Step 1. Revise long multiplication with two figures in top line.

Step 2. Increase top line to 3 digits—do not increase multiplier beyond 2 digits so that maximum point would not go beyond 999×99.

SHORT DIVISION

Step 1. Increase the number to be divided to 4 figures; include 0 difficulties, learnt in Class 3, in the later sums to be done by the class.

Step 2. Increase the number to be divided to 5 figures.

LONG DIVISION

Step 1. Divide numbers that are multiples of 10 by 20, 30, 40 with no remainder and a single unit answer.

(*a*) Put the following sum on the blackboard:

$$80 \div 20$$

(*b*) Show how the method of setting out is different from the method done previously, because the division is by a number over 12; it is done as follows (on the blackboard):

$$20\overline{)80}$$

(*c*) Ask the class:

'How many 20s in 8?'

When it is stated that there are none, emphasise to the class that it *now* asks itself 'How many 20s in 80?' and that the 8 is *not* left out.

(*d*) Say:

'The first thing we did was to *divide* 80 by 20' so put the word *divide* on the blackboard to the right of the sum.

Note carefully:

When the class was asked how many 20s in 80, it will have given the answer 4. Show how it is placed over the *second* figure of the number being divided, pointing out that as 20 does not go into 8, there is no figure to be placed over the 8.

Now multiply the 20 by 4 and place the 80 in its place under the dividend. Point out that this is the second step and write the word 'Multiply' under the word 'Divide' on the right side.

The blackboard now reads as

$$\begin{array}{rl} 4 & \\ 20\overline{)80} & \quad \text{Divide} \\ 80 & \quad \text{Multiply} \end{array}$$

Explain that it is necessary to find how much is left over so we must take away (subtract). Do this with the class and write the word 'Subtract' on the right side under 'Multiply', pointing out that this is the third step. The blackboard reads

$$\begin{array}{rl} 4 & \\ 20\overline{)80} & \quad \text{Divide} \\ \underline{80} & \quad \text{Multiply} \\ \text{—} & \quad \text{Subtract} \end{array}$$

The teacher will go over the method of doing the sum again, referring to the three steps written on the right.

Step 2. Division of 2 digit numbers with no remainder by numbers between 13 and 19

Do the following sum with the children on the blackboard.

6	
13)78	Divide
78	Multiply
—	Subtract

Give 10 sums like this.

Step 3. Division of 3 digit numbers, by 13 only, in which the process of bringing down is taught.

The teacher puts this sum on the blackboard.

13)143

and with the class does it as far as the point shown here (which has been done in Step 1).

1	
13)143	Divide
13	Multiply
1	Subtract

He now points out that only part of the dividend has been divided, a 10 and 3 units still remaining.

Show how the 3 is brought down and placed after the 1 left over in the ten column to make the number 13. He says: 'We can now go forward and complete the sum by dividing by 13.' So the blackboard now reads

11	*Ans.*	
13)143		Divide
13		Multiply
13	Subtract, Bring Down,	Divide
13		Multiply
—		Subtract

Give at least 10 sums like this for practice (no remainders).

Step 4. Division of 3 digit numbers by 14, 15 and so on to 19. There must be no remainder.

The teacher will give further sums with these divisions, having done one with the class on the blackboard so that they see they are similar. He must be sure that the first 2 digits of the number to be divided are not less than the divisor; e.g.

$$18\overline{)198} \qquad 14\overline{)224} \qquad 15\overline{)180} \qquad 15\overline{)285}$$

Step 5. Division of 3 digit number with no remainders by numbers up to 49. The teacher will again do one example with the class such as

$$28\overline{)588}$$

and then give at least 10 sums like them.

Step 6. Division without remainders of 3 digit numbers with first 2 digits of dividend less than the divisor.

The teacher does the following sum on the blackboard:

$$18\overline{)162}$$

He will teach carefully that as when we find 18 does not go into 1 we have a blank space in the answer space over the 1, so when it is found 18 does not go into 16 a blank space is also left over the 16; that it must then be asked: 'How many 18's in 162?'

Give at least 10 examples of these sums.

Step 7. Division without remainders of 4 digit numbers. (*a*) Divisors will be low at first, gradually increasing to 49, such as

$$17\overline{)1961} \qquad 22\overline{)2904}$$

Give at least 10 sums like this.

(*b*) Then on the blackboard do a sum like those in Step 6, e.g.

$$32\overline{)2688}$$

Give at least 10 sums with those of (*a*) and (*b*) mixed up.

Step 8. Introduce remainders. Start with low dividends and gradually increase them.

Do the following sum on the blackboard with the class :

```
   21 r 1  Ans.
14)295
   28
   --
    15
    14
    --
     1 r
    --
```

Give at least 10 sums like these.

Step 9. Division of 4 digits with a medial zero in the answer.

$$16\overline{)3264}$$

Give 10 sums like this.

Step 10. Division by divisors up to 99 without remainders first and remainders afterwards.

MONEY

ADDITION

Aim

To increase gradually the number of shillings until the total column reaches sh. 999.99 with a maximum of 5 items.

(i)

sh.	ct.
29	91
57	67
+68	85

sh.	ct.
271	60
18	75
+194	18

(ii)

sh.	ct.
37	63
47	16
24	79
+ 9	18

sh.	ct.
100	99
77	43
106	17
+ 8	72

sh.	ct.
7	04
18	73
97	00
+472	49

sh.	ct.
444	58
107	29
41	07
+225	55

(iii)

sh.	ct.		sh.	ct.
15	75			59
175	79		290	21
8	82		7	16
706	71		301	78
+ 11	48		+127	34

Proceed with examples of these types, giving many examples.

SUBTRACTION

Aim

To increase gradually the number of shillings in the top line to sh. 999.99.

sh.	ct.		sh.	ct.		sh.	ct.
95	61		106	05		264	26
−71	75		− 49	67		−194	32

sh.	ct.		sh.	ct.		sh.	ct.
493	17		720	34		998	00
−217	48		−667	53		−743	27

Proceed with examples of this kind, giving plenty of practice.

PROFIT AND LOSS

Step 1. *Definitions.*

First teach the meaning of the terms used in this topic.

Profit is the amount of money a person gains by selling an article for more than the price he paid for it. E.g. if a man bought a chair for sh. 4.00 and sold it for sh. 4.50, his profit was 50 ct. Notice, the profit does *not* include the price he paid.

Loss is the amount of money a person loses by selling an article for less than the price he paid for it. E.g. if a man bought a chair for sh. 4.00 and sold it for sh. 3.50 his loss was 50 ct. He lost 50 ct.

Cost Price, which is usually written C.P., is the amount paid for the article when the man first bought it.

Selling Price is the price received for the article when the same man sells it.

E.g. in the above example of Profit:

Cost Price (C.P.) is sh. 4.00.
Selling Price (S.P.) is sh. 4.50.

In the above example of Loss:

Cost Price (C.P.) is sh. 4.00.
Selling Price (S.P.) is sh. 3.50.

Step 2. *Formula.*

The children should be led by the teacher, through several examples like these quoted above, to formulate the rule:

$$\text{Profit} = \text{S.P.} - \text{C.P.}$$
$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

Much oral work must be done to ensure that the children fully understand the process before being introduced to the written statements. Mental work should be graded from using simple numbers to those using more difficult numbers which the children are capable of handling mentally.

Profit e.g. (*a*) A pot was bought for sh. 1 and sold for sh. 2; how much profit was made?
(*b*) A basket was bought for sh. 2.50 and sold for sh. 3; how much profit was made?
(*c*) A mat was bought for sh. 6.50 and sold for sh. 8; how much profit was made?
(*d*) Two hens were bought for sh. 2 each and sold for sh. 3 each; how much profit was made altogether? etc.

Loss e.g. (*a*) A man bought a jembe for sh. 6 and sold it for sh. 5; what was the loss?
(*b*) A pot was bought for sh. 1.50 and sold for sh. 1; what was the loss? etc.

Step 3. *Written Work and Practice.*

When the definitions and formulae are understood, demonstrate the method of setting down as follows :

Example of Profit (as above) :

C.P. =sh. 4.00
S.P. =sh. 4.50
Profit=S.P. − C.P.
=sh. 4.50 − sh. 4.00
=50 ct.

Example of Loss (as above) :

C.P. =sh. 4.00
S.P. =sh. 3.50
Loss=C.P. − S.P.
=sh. 4.00 − sh. 3.50
=50 ct.

Give written work:

Profit e.g. 1. John bought beans for sh. 5.50 and sold them for sh. 7. How much profit did he make?

e.g. 2. Mary bought a stool for sh. 3 and sold it for sh. 3.50. How much profit did she make?

Loss e.g. 3. A hen was bought for sh. 4.50 and sold for sh. 3.50. What was the loss?

e.g. 4. Bananas were bought for sh. 8.50 and sold for sh. 6.25. What was the loss?

Give 10 more examples of Profit and 10 of Loss, increasing the difficulty.

FRACTIONS

Aim

Introduction of $\frac{1}{12}$, $\frac{1}{5}$ and $\frac{1}{9}$: addition and subtraction: improper and proper fractions: addition of 3 items.

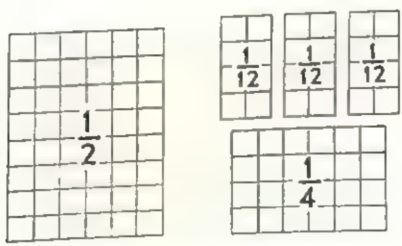
Step 1. Introduction of twelfths ($\frac{1}{12}$).

This step will itself provide a revision of all of the main steps carried out in Class 3, and separate introductory revision is not, therefore, necessary. Care should be taken, however, to see that the children (and not the teacher) provide all the information for steps which have already been dealt with in Class 3, or which are merely a projection of steps learnt there. Only when a completely new concept is introduced should it become necessary for the teacher to take the lead.

(*a*) Each child is given 2 pieces of paper exactly as in Class 3, the first one being numbered by all children as a whole unit.

(*b*) The second one is then divided and numbered by stages into:

(i) One half, (ii) one quarter, (iii) three one-twelfth pieces as follows:



(*c*) From this is built up the following understanding by the usual stages and practical work:

(i) Three $\frac{1}{12}$ths equal $\frac{1}{4}$.
(Revision)

(ii) How many $\frac{1}{4}$s in a whole? (4)

(iii) If there are three $\frac{1}{12}$ths in $\frac{1}{4}$ and four $\frac{1}{4}$s in 1, there are twelve $\frac{1}{12}$ths in one whole.

(iv) Six $\frac{1}{12}$ths in a half.

At this stage each child should be issued as additional equipment with a simplified foot-rule (as used in Class 2 for linear measure) to use in addition to his paper pieces. The foot-rule has the advantage of being marked off in twelve $\frac{1}{12}$ segments whereas the paper pieces provide an easy comparison with quarters and halves. Thus both will be used for working, either separately or in conjunction, as the occasion requires.

Step 2. Addition of fractions, showing the need of a Common Denominator.

Put on the blackboard :

$$\tfrac{1}{2}+\tfrac{1}{4}$$

Elicit the answer :

$$\tfrac{1}{2}+\tfrac{1}{4}=\tfrac{3}{4}$$

Point out we are able to do this because we have been cutting paper and have found the answer in that way and have remembered it.

Proceed to show that we can get the answer by working the sum from our knowledge of fractions. Ask how many $\frac{1}{4}$s in a $\frac{1}{2}$, putting on the blackboard :

$$\tfrac{1}{2}+\tfrac{1}{4}$$
$$=\tfrac{2}{4}+\tfrac{1}{4}$$

Ask the class 'What is the total of 2 cows and 1 cow?' (3 cows). Now point out that our answer on the blackboard resembles this sum, that we can say 2 quarters and 1 quarter equals 3 quarters or

$$\tfrac{2}{4}+\tfrac{1}{4}=\tfrac{3}{4}$$

By bringing our original fraction of $\frac{1}{2}+\frac{1}{4}$ to quarters, or *to the same family*, it was easy to add our fractions. As this is the essential principal of the Common Denominator, the teacher must ensure that he makes this step clear with its idea of a 'common family'. The teacher must ensure that the equals sign is clearly and correctly used right from the beginning.

Step 3. To give further practice in understanding Step 2. Put on the blackboard :

$$\tfrac{1}{2}+\tfrac{1}{8}$$

Ask for an answer. Some children may give it. Perhaps most will fail. Now say ' Let us bring the fractions to the same family.

' How many $\tfrac{1}{8}$s in a $\tfrac{1}{2}$? ' (4)

Write on blackboard, under the sum :

$$\tfrac{1}{2}+\tfrac{1}{8}$$
$$=\tfrac{4}{8}+\tfrac{1}{8}$$

and elicit the answer :

$$=\tfrac{5}{8}$$

Point out that $\tfrac{1}{8}$s were chosen as the common family, not $\tfrac{1}{2}$s, because there are no $\tfrac{1}{2}$s in an $\tfrac{1}{8}$.

Step 4. Consolidate the understanding of this principle with yet another example, done in the same way, e.g. $\tfrac{1}{4}+\tfrac{5}{12}$.

Step 5. Show the mechanical method of finding the Common Denominator, and of working the sum. Put the sum of Step 3 down on the blackboard again.

$$\tfrac{1}{2}+\tfrac{1}{8}$$

Point out that it is easy to find the common family to which both fractions belong by asking ourselves :

' What is the lowest figure which will hold the 2 and the 8 an exact number of times? ' (No remainder).

What is it? 8 (or the $\tfrac{1}{8}$th family).

(If there are some children appearing not to understand, explain 2 goes into 8, 4 times ; 8 goes into 8, once.)

So we write on the blackboard under the sum :

$$\tfrac{1}{2}+\tfrac{1}{8}$$
$$=\tfrac{\ }{8}+\tfrac{\ }{8}$$

How many $\tfrac{1}{8}$s in a $\tfrac{1}{2}$? ($\tfrac{4}{8}$)

Point out here that we could get the same answer by finding 2 goes into 8, 4 times ; 4 times 1 is 4.

or $\quad 2\overline{)8}$

$\quad\quad 4 \qquad 4\times 1=4$

So the sum so far reads

$$\tfrac{1}{2}+\tfrac{1}{8}$$
$$=\tfrac{4}{8}+\tfrac{\ }{8}$$

Now repeat this with

$$8\overline{)8}\qquad$$
$$1 \qquad 1\times1=1$$

and put on blackboard

$$\tfrac{1}{2}+\tfrac{1}{8}$$
$$=\tfrac{4}{8}+\tfrac{1}{8}$$
$$=\tfrac{5}{8}$$

Bring to the notice of the children that we know $\frac{4}{8}=\frac{1}{2}$, and we have also found this answer even if we do *not* know the answer mentally, and also that this is a very quick mechanical way of finding an answer that would take a longer time to work mentally in some longer fractions we will be doing later.

Step 6. Repeat the previous step with the following example:

$$\tfrac{1}{3}+\tfrac{1}{4}$$

Put emphasis on the question 'What is the lowest figure to hold 3 and 4?', and make sure all children understand this.

When the sum is completed show that, having found the Common Denominator, one long line is more suitable, so the blackboard reads:

$$\frac{1}{3}+\frac{1}{4} \qquad\qquad \frac{1}{3}+\frac{1}{4}$$
$$=\frac{4}{12}+\frac{3}{12} \qquad =\frac{4+3}{12}$$
$$=\frac{7}{12} \qquad\qquad =\frac{7}{12}$$

At this point introduce the term Common Denominator emphasising that it is a term meaning a common family.

Since this sum has been done purely mechanically for the first time, it is advisable to make the children do a practical working of it, so they may see how quickly they have solved the sum in this way.

Step 7. Subtraction of fractions.

Put on blackboard:

$$\tfrac{1}{2}-\tfrac{1}{4}$$

Elicit the answer:

$$\tfrac{1}{2}-\tfrac{1}{4}=\tfrac{1}{4}$$

Say to the children 'We will do the sum in the way we have just learnt as if we could not work it mentally'. Do it with the class so that the blackboard reads

$$\frac{1}{2}-\frac{1}{4}$$

$$=\frac{2-1}{4}$$

$$=\frac{1}{4}$$

Point out that the sum is not different from those learnt before except that taking away takes the place of adding.

Give the following exercises:

(*a*) $\tfrac{1}{2}+\tfrac{1}{3}$; $\tfrac{1}{4}+\tfrac{1}{6}$; $\tfrac{1}{2}+\tfrac{1}{12}$; $\tfrac{1}{4}+\tfrac{1}{12}$; $\tfrac{1}{3}+\tfrac{1}{12}$;
$\tfrac{1}{2}+\tfrac{1}{6}$; $\tfrac{1}{3}+\tfrac{1}{6}$; $\tfrac{1}{12}+\tfrac{1}{6}$; $\tfrac{1}{3}+\tfrac{1}{4}$.

(*b*) $\tfrac{1}{2}-\tfrac{1}{3}$; $\tfrac{1}{4}-\tfrac{1}{6}$; $\tfrac{1}{2}-\tfrac{1}{12}$; $\tfrac{1}{4}-\tfrac{1}{12}$; $\tfrac{1}{3}-\tfrac{1}{12}$;
$\tfrac{1}{2}-\tfrac{1}{6}$; $\tfrac{1}{3}-\tfrac{1}{6}$; $\tfrac{1}{6}-\tfrac{1}{12}$.

Note carefully:

At this stage, do *not* attempt to require children to bring fractions to their lowest terms; e.g. when $\tfrac{4}{12}$ is the answer, do not ask children to reduce it to $\tfrac{1}{3}$. Both are to be accepted.

Step 8. Fractions having numerators above 1.

Put on the blackboard $\dfrac{1}{8}+\dfrac{3}{4}$

What is the Common Denominator?

Elicit the answer: blackboard reads $\dfrac{\quad}{8}$

Elicit the top line: blackboard reads $\dfrac{1+6}{8}$

It is probable that many children will have difficulty in arriving at the figure 6. It is advisable here to ask 'How many $\tfrac{1}{8}$s in $\tfrac{1}{4}$?'

(2) 'How many $\frac{1}{8}$s in $\frac{3}{4}$?' (6)

Point out that we have got that answer. Then show how we can get it by the mechanical method of Step 5; that is

$$4\overline{)8} \qquad \underline{2} \qquad 2 \times 3 = 6$$

Put on the blackboard

$$\tfrac{1}{2} + \tfrac{3}{8}$$

Do this on the blackboard with the class, getting the class to do the sum and ensuring that every child is understanding. Do one more example with subtraction; if necessary, for slower children, do further examples on the blackboard.

Give the following exercises:

(*a*) $\frac{3}{8}+\frac{1}{4}$; $\frac{1}{6}+\frac{3}{4}$; $\frac{1}{2}+\frac{3}{12}$; $\frac{1}{8}+\frac{1}{12}$; $\frac{5}{8}+\frac{1}{4}$;
$\frac{3}{8}+\frac{1}{12}$; $\frac{2}{3}+\frac{1}{6}$; $\frac{3}{8}+\frac{1}{2}$; $\frac{1}{4}+\frac{7}{12}$.

(*b*) $\frac{5}{6}-\frac{1}{3}$; $\frac{2}{3}-\frac{1}{6}$; $\frac{7}{8}-\frac{1}{4}$; $1-\frac{1}{6}$; $1-\frac{5}{8}$;
$1-\frac{2}{3}$; $\frac{7}{8}-\frac{1}{2}$; $\frac{5}{6}-\frac{1}{8}$; $\frac{2}{3}-\frac{1}{4}$.

(*c*) $\frac{3}{8}+\frac{5}{12}$; $\frac{2}{3}+\frac{1}{4}$; $\frac{3}{4}-\frac{3}{8}$; $\frac{7}{8}-\frac{5}{12}$; $\frac{2}{3}-\frac{7}{12}$;
$\frac{3}{4}-\frac{2}{3}$; $\frac{7}{8}-\frac{2}{3}$; $\frac{2}{3}-\frac{5}{8}$.

Step 9. Reduction of fractions to their lowest terms.

(*a*) Put on the blackboard

$$\tfrac{1}{4}+\tfrac{1}{4}=\tfrac{2}{4}$$

'Could we have another answer?' ($\frac{1}{2}$)

Blackboard reads as $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$.

(*b*) Put on the blackboard

$$\tfrac{1}{8}+\tfrac{1}{8}=\tfrac{2}{8}$$

'Could we have another answer?' ($\frac{1}{4}$)

Blackboard reads as $\frac{1}{8}+\frac{1}{8}=\frac{2}{8}=\frac{1}{4}$.

Now refer back to example (*a*).

Ask which is the more sensible fraction to have in the answer; elicit $\frac{1}{2}$ because it is more readily understood (we speak about $\frac{1}{2}$ an orange, not $\frac{2}{4}$) and because it is in its simplest form. Proceed to show that it is easy to reduce a fraction to its simplest form by finding and dividing by the figure which divides equally into the

top and bottom number. So 2 divides equally into 2 and 4 and we have

$$\frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}}=\frac{1}{2}$$

Deal in the same way with example (*b*).

Put a third example on the blackboard.

$$\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}=\tfrac{3}{4}$$

Ask the class if this answer is in its simplest form. Use this example to emphasise that only if the *same* figure divides equally into top and bottom numbers we reduce a fraction; if there is no such figure, then the fraction is already in its lowest terms. The teacher must make sure that this is firmly understood at this point.

The class will then do the following exercise, reducing the fractions to the lowest terms.

1. $\frac{4}{6}$, $\frac{3}{6}$, $\frac{2}{8}$, $\frac{6}{8}$, $\frac{6}{12}$, $\frac{2}{6}$, $\frac{3}{12}$.
2. $\frac{6}{24}$, $\frac{9}{12}$, $\frac{6}{18}$, $\frac{10}{12}$, $\frac{4}{12}$, $\frac{8}{24}$, $\frac{12}{24}$.
3. $\frac{3}{9}$, $\frac{5}{15}$, $\frac{4}{16}$, $\frac{9}{18}$, $\frac{16}{24}$, $\frac{12}{18}$, $\frac{9}{12}$, $\frac{6}{9}$, $\frac{3}{24}$.

Step 10. Addition and subtraction with answers in the lowest terms.

Put on the blackboard

$$\frac{1}{3}+\frac{1}{6}$$

The teacher does this with the class, so the blackboard reads

$$\frac{1}{3}+\frac{1}{6}=\frac{2+1}{6}=\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}}}=\frac{1}{2}$$

Explain that the cancellation of the fraction to the lowest terms must be done (and done neatly).

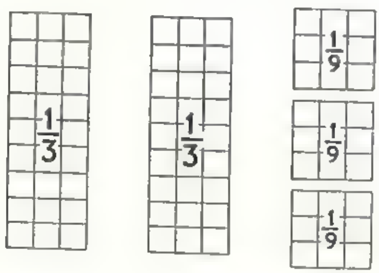
Give the following exercise:

$\frac{1}{2}+\frac{1}{3}$; $\frac{3}{4}+\frac{1}{12}$; $\frac{11}{12}-\frac{1}{4}$; $\frac{7}{12}-\frac{1}{4}$; $\frac{7}{12}+\frac{1}{4}$;
$\frac{3}{4}-\frac{3}{8}$; $\frac{5}{12}+\frac{1}{3}$; $\frac{6}{12}+\frac{1}{4}$; $\frac{7}{12}-\frac{1}{3}$.

Step 11. Introduction of new fractional parts.

A. $\frac{1}{9}$.

(i) This is a logical development from thirds, and has not been dealt with previously, purely as a matter of convenience. It should be possible to introduce this purely in theory by now, but if backward children have difficulty, take them in a separate group with pieces of paper, dividing thirds into 3 pieces as follows:



(*Note:* Do not use the standard-sized pieces of paper used previously, but others with a multiple of 9 squares on them.)

(ii) Simple exercises on ninths.

(*a*) $\frac{1}{3}+\frac{1}{9}$; $\frac{2}{3}+\frac{1}{9}$; $\frac{1}{3}-\frac{1}{9}$; $\frac{2}{3}-\frac{1}{9}$; $1-\frac{1}{9}$;
$\frac{2}{9}+\frac{1}{3}$; $\frac{4}{9}-\frac{1}{3}$; $\frac{2}{3}-\frac{4}{9}$; $\frac{5}{9}+\frac{1}{3}$; $\frac{2}{3}-\frac{2}{9}$.

(*b*) Related to other simple fractional parts (L.C.D. practice).
$\frac{1}{2}+\frac{1}{9}$; $\frac{1}{4}-\frac{1}{9}$; $\frac{4}{9}+\frac{1}{4}$; $\frac{3}{4}-\frac{2}{9}$; $\frac{3}{4}+\frac{1}{9}$;
$\frac{1}{4}+\frac{5}{9}$; $\frac{1}{2}-\frac{2}{9}$; $\frac{3}{4}-\frac{5}{9}$; $\frac{4}{9}-\frac{1}{4}$; $\frac{7}{9}-\frac{1}{2}$.

B. $\frac{1}{10}$ and $\frac{1}{5}$.

(i) A practical introduction to this is best done with each of the children referring to their manufactured rulers (not the home-made foot). Tenths can easily be read off from this down one side of the ruler, and it should be easy to build up to fifths from this straight away.

(ii) Simple practical work.

(*a*) $\frac{1}{10}+\frac{1}{5}$; $\frac{2}{5}+\frac{3}{10}$; $1-\frac{3}{10}$; $\frac{3}{5}+\frac{3}{10}$; $1-\frac{2}{5}$;
$\frac{3}{5}-\frac{3}{10}$; $\frac{7}{10}+\frac{1}{5}$; $\frac{9}{10}-\frac{3}{5}$; $\frac{4}{5}-\frac{7}{10}$; $\frac{7}{10}-\frac{1}{5}$.

(*b*) Relations to other simple fractional parts (and L.C.D. practice again of course).

$\frac{1}{2}+\frac{1}{5}$; $\frac{1}{2}+\frac{1}{10}$; $\frac{3}{5}-\frac{1}{3}$; $\frac{7}{10}+\frac{1}{4}$;
$\frac{4}{5}-\frac{2}{3}$; $\frac{3}{5}-\frac{1}{3}$; $\frac{7}{10}-\frac{1}{3}$; $\frac{1}{10}+\frac{3}{4}$; $\frac{2}{5}-\frac{1}{4}$;
$\frac{9}{10}-\frac{1}{3}$; $\frac{3}{4}+\frac{1}{5}$; $\frac{2}{3}-\frac{3}{10}$; $\frac{3}{4}-\frac{7}{10}$; $\frac{1}{2}-\frac{2}{5}$.

(*c*) With the new parts now introduced we have covered all of the fractional parts necessary for use in the primary school. The introduction of tenths provides the starting points for the introduction of decimals, and for further fractional work we now have available denominators of 2, 3, 4, 5, 6, 8, 9, 10, and 12.

It is neither necessary nor advisable to introduce $\frac{1}{7}$ or $\frac{1}{11}$ in primary classes as these parts add nothing to the understanding of fractions as a whole and merely tend to lead to unduly complicated common denominators at the present stage of the pupils' understanding of fractions.

Step 12. In order to consolidate work up to this point, the teacher will give further exercises, observing carefully the following rules :

1. Only fractional parts with denominators 2, 3, 4, 5, 6, 8, 9, 10, and 12 to be used. Sometimes use the whole unit 1 at the beginning of a subtraction.

2. *Not* more than 2 items in a sum.

3. Give a good mixture of both addition and subtraction.

4. *No* answers to give more than 1 whole unit.

Step 13. Introduction of Improper Fractions.

First show how to deal with Improper Fractions in the answers.

(*a*) Put on the blackboard $\frac{1}{2}+\frac{3}{4}$
Elicit the answer $=\frac{5}{4}$
Ask ' How many $\frac{1}{4}$s in a whole? ' (4)
Put on the blackboard $\frac{4}{4}$
and point out that we have more quarters than this in the answer of our sum.

Ask if there is a whole number in the answer to our sum and put on the blackboard $\quad 1$

Then elicit that there is $\frac{1}{4}$ left over $\quad 1$ and $\frac{1}{4}$

Show that we write this as $\quad 1\frac{1}{4}$

(*b*) Put on the blackboard $\quad \frac{1}{2}+\frac{5}{8}$

Elicit the answer mentally or worked out on the blackboard $\quad =\frac{9}{8}$

Ask how many $\frac{1}{8}$s in 1? (8)

Elicit as in (*a*) that there is a whole unit 1 in $\frac{9}{8}$ and $\frac{1}{8}$ over, so finally the blackboard reads

$$\frac{1}{2}+\frac{5}{8}$$
$$=\frac{9}{8}$$
$$=1\frac{1}{8}$$

(*c*) Put on the blackboard $\quad \frac{2}{3}+\frac{5}{6}$

Work it out with the class on the blackboard as

$$\frac{4+5}{6}$$
$$=\frac{9}{6}$$

Point out that it is still required to bring fractions to the lowest terms as here

$$=\frac{3}{2}$$
$$=1\frac{1}{2} \quad \textit{Ans.}$$

Teach the terms Proper Fraction and Improper Fraction.

Give exercises as follows:

$\frac{5}{4}+\frac{5}{6}$; $\quad \frac{7}{8}+\frac{1}{2}$; $\quad \frac{3}{5}+\frac{1}{2}$; $\quad \frac{5}{9}+\frac{2}{3}$; $\quad \frac{5}{8}+\frac{3}{4}$;
$\frac{7}{12}+\frac{2}{3}$; $\quad \frac{5}{6}+\frac{5}{8}$; $\quad \frac{2}{5}+\frac{7}{10}$; $\quad \frac{3}{5}+\frac{2}{3}$.

Step 14. Show how to deal with whole numbers in the sum to be worked.

(*a*) Put on the blackboard $\frac{1}{2}+1\frac{1}{3}$

Point out that we have now a whole number fraction or Proper Fraction in our sum. Invite the class to do this sum on the blackboard. It is possible and even probable that some children will realise that if the Proper Fraction is converted to an Improper Fraction they will be left with a sum that is not different from those they have learnt to do. Let a child do this if there is a volunteer. Then do it with the class.

Ask the class how many $\frac{1}{3}$s in $1\frac{1}{3}$ and then write on the blackboard

$$\frac{1}{2}+1\frac{1}{3}$$
$$=\frac{1}{2}+\frac{4}{3}$$

Point out to the class that this sum is now similar to all others they have done and complete it with the class.

(*b*) Put on the blackboard $1\frac{5}{8}+\frac{2}{8}$

Ask the class to supply the next line. It is probable some children will be slow to work out that

$$1=\frac{8}{8}. \quad \frac{8}{8}+\frac{5}{8}=\frac{13}{8}$$

However, give the slower children time and when most or all are ready to answer, accept them. Now point out how slow some children were to get the answer, so the teacher will show a quick method. The teacher then asks how many $\frac{1}{8}$s in a 1; when told 8, he will show that if we multiply the whole unit 1 by the denominator 8 we get that answer. $1\times 8=8$ (or $\frac{8}{8}$)

Then all we need do is add on the rest of the fraction, or the numerator, to change the Proper Fraction into an improper one $\frac{8}{8}+\frac{5}{8}=\frac{13}{8}$

Finish the blackboard sum.

(*c*) At this point do some blackboard examples of changing Proper into Improper Fractions:

$1\frac{3}{8}$; $1\frac{3}{4}$; $2\frac{5}{6}$; $2\frac{7}{8}$; $3\frac{3}{8}$, etc.

(*d*) It will be noted that in Step 14, exercises were done in which answers had to be changed from Improper Fractions into Proper Fractions by reasoning only. They were very easy sums. The purely mechanical method has not yet been shown. This is a convenient place, as it follows the reverse process of (*c*) in this step.

Put on the blackboard $\frac{11}{8}$

Ask the class to change it into a Proper Fraction $1\frac{3}{8}$

Point out that we can do it mechanically by saying how many 8's in 11? (1 and 3 over)

So $\therefore$ 1 whole and $\frac{3}{8}$ over $=1\frac{3}{8}$

Give the following exercise, changing into Proper Fractions.

$\frac{15}{8}$; $\frac{16}{8}$; $\frac{11}{5}$; $\frac{9}{4}$; $\frac{17}{10}$; $\frac{27}{12}$, etc.

(*e*) Do another example on the blackboard, children to do the work. $1\frac{3}{5}+1\frac{7}{10}$

Give exercises as follows:

$1\frac{3}{4}+2\frac{1}{3}$; $1\frac{1}{2}+2\frac{1}{6}$; $\frac{7}{8}+2\frac{1}{3}$; $2\frac{3}{5}+1\frac{3}{10}$; $3\frac{1}{4}+1\frac{1}{3}$; $2\frac{5}{12}+2\frac{1}{3}$.

Give 30 examples at least.

Note that:

(i) Not more than 3 in the whole units to be given.

(ii) Only the fractional parts learned are to be used.

(iii) Whole units are *always* to be changed into improper fractions as the first step.

Step 15. Improper Fractions: Subtraction.

Put on the blackboard $2\frac{11}{12}-1\frac{1}{4}$

Tell the children that the rules are exactly the same as for addition and go through them as follows:

(i) *Always* change whole units into improper fraction $=\frac{35}{12}-\frac{5}{4}$

(ii) Find the Common Denominator and proceed with subtraction on the top line $=\dfrac{35-15}{12}$

$=\frac{20}{12}$

(iii) *Always* reduce the final fraction to its lowest terms if it will cancel $=\frac{5}{3}$

(iv) Always change the answer back into whole units if the numerator is larger than the denominator $=1\frac{2}{3}$ *Ans.*

Point out here that

(*a*) Cancelling should always be done before converting the fraction into a proper fraction, so:

Right $\dfrac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{12}}}=\dfrac{5}{3}=1\frac{2}{3}$

Wrong $$\frac{20}{12}=1\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}}=1\tfrac{2}{3}$$

(*b*) Try to cancel once, rather than more often, so

$$\frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{12}}} \text{ is better than } \frac{\overset{\overset{5}{\cancel{10}}}{\cancel{20}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{12}}}$$

(*c*) Cancel the whole number, not each figure, separately and write the new figures *either* directly above *or* to the right whichever is more convenient, so

$$\frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{12}}} \text{ or } \frac{\cancel{20}\,5}{\cancel{12}\,3} \quad \textit{not} \quad \frac{\cancel{20}\,5}{\underset{3}{\cancel{12}}}$$

Give exercises as follows :

$$2\tfrac{1}{4}-\tfrac{5}{8} \qquad 3\tfrac{5}{6}-\tfrac{3}{4} \qquad 2\tfrac{1}{5}-\tfrac{2}{3}$$
$$3\tfrac{1}{12}-1\tfrac{1}{2} \qquad 3\tfrac{2}{3}-2\tfrac{1}{2} \qquad 4\tfrac{1}{12}-2\tfrac{5}{6} \qquad \text{etc.}$$

Give at least 20 examples (do not go·beyond 5 in any whole unit).

Step 16. Introduction of 3 items in addition.

This should present no difficulty at this stage. The teacher demonstrates the following sum on the blackboard with the children's participation :

$$\begin{aligned} &1\tfrac{1}{4}+\tfrac{3}{8}+\tfrac{1}{3}\\ =&\tfrac{5}{4}+\tfrac{3}{8}+\tfrac{1}{3}\\ =&\frac{30+9+8}{24}\\ =&\tfrac{47}{24}\\ =&1\tfrac{23}{24}\ \textit{Ans.}\end{aligned}$$

Some children may find difficulty at first, now that there are

3 denominators for which to find a common denominator. Quick practice with the class, finding common denominators for

3, 6, 12	3, 4, 8	3, 4, 6	
4, 6, 8	6, 8, 12	2, 5, 6	
2, 5, 10	4, 5, 10	2, 4, 10	etc.

will soon clear this up.

Give exercises as follows:

$$\tfrac{4}{5}+\tfrac{1}{2}+\tfrac{3}{10} \qquad \tfrac{3}{4}+\tfrac{2}{3}+\tfrac{5}{6}$$
$$\tfrac{5}{8}+\tfrac{3}{4}+\tfrac{2}{3} \qquad \tfrac{7}{8}+\tfrac{7}{10}+\tfrac{4}{5}$$
$$1\tfrac{2}{3}+\tfrac{5}{8}+\tfrac{3}{4} \qquad \tfrac{11}{12}+1\tfrac{1}{4}+\tfrac{2}{3}$$

etc.

Give at least 25 sums like these (in order to prevent too much unnecessary complex working at this stage, do not give any item over 2 whole units).

Teachers should not teach or allow any shortened forms of working fractions in this class.

MULTIPLICATION OF MONEY

SHORT MULTIPLICATION

Aim

To proceed from the stage reached in Class 3, increasing figures gradually in the shillings column of the sum multiplied to a maximum of sh. 999.

LONG MULTIPLICATION

Aim

The multiplying of sh. ct. (maximum figure of 99 in the sh. column) by 2 figures.

Step 1. Multiplying sh. ct. by multiples of 10, one figure only in the sh. column. The working must be done below the line as shown.

sh.	ct.		sh.	ct.	
3	25		8	36	
	×20			×30	
65	00	*Ans.*	250	80	*Ans.*
5	500		10	1080	
60			240		
65			250		

Give at least 10 sums like these.

Step 2. 2 figures in the sh. column.

sh.	ct.		sh.	ct.	
13	25		28	36	
	×20			×30	
265	00	*Ans.*	850	80	*Ans.*
5	500		10	1080	
260			840		
265			850		

Give at least 10 sums like these.

Step 3. Proceed to multiplying by figures 13 to 19 only. Again start with one figure in sh. column. Ensure that children do multiply by the unit figure of the top line, that is, that they say 6 times 13, not 13 times 6.

sh.	ct.		sh.	ct.	
6	32		9	41	
	×13			×14	
82	16	*Ans.*	131	74	*Ans.*
4	96		5	164	
78	320		126	410	
82	416		131	574	

Give at least 10 sums like these.

Step 4. 2 figures in sh. column.

sh.	ct.		sh.	ct.	
14	72		23	84	
	×15			×16	
220	80	*Ans.*	381	44	*Ans.*
10	360		13	504	
70	720		138	840	
140	1080		230	1344	
220			381		

Give 10 sums like these.

Step 5. Increase the multiplier by stages up to 99. Be careful to use a multiplier that does not give an answer over sh. 999.

DIVISION OF MONEY

SHORT DIVISION

Aim

Short division will be carried on from stage reached in Class 3. Increase the dividend to sh. 999.99 gradually. The insertion of the carrying figure will be stopped as each child appears able to do without it.

LONG DIVISION

Aim

Division by 2 digits, dividend not to exceed sh. 999.99.

Step 1. Long division of sh. ct. by multiples of 10 only, with no remainder.

	sh.	ct.			sh.	ct.	
	4	03	*Ans.*		3	00	*Ans.*
20)	80	60		40)	120	00	
	80	60			120		
	—	—			—		

Give 10 sums like these.

Step 2. Division of sh. ct. by multiples of 10, introducing the method of bringing down and subtracting, but no carrying. Recall the method in Long Division of Number.

sh.	ct.	
15	03	*Ans.*
30)450	90	
30	90	
150	—	
150		
—		

sh.	ct.	
22	01	*Ans.*
40)880	40	
80	40	
80	—	
80		
—		

Give 10 sums like these.

Step 3. As Step 2, but with carrying from sh. to ct.

sh.	ct.	
15	48	*Ans.*
30)464	40	
30	1400	
164	1440	
150	120	
14	240	
× 100	240	
1400 ct.	—	

sh.	ct.	
15	71	*Ans.*
40)628	40	
40	2800	
228	2840	
200	280	
28	40	
× 100	40	
2800 ct.	—	

Give at least 10 sums like these.

Step 4. Division by numbers 13 to 19, with no remainders.

sh.	ct.	
3	51	*Ans.*
13)45	63	
39	600	
6	663	
× 100	65	
600 ct.	13	
	13	
	—	

sh.	ct.	
19	32	*Ans.*
18)347	76	
18	500	
167	576	
162	54	
5	36	
× 100	36	
500 ct.	—	

Give 10 sums like these.

Step 5. Introduce sums as for Step 4, but with remainders.

sh.	ct.
3	51 r 4 ct. *Ans.*
13)45	67
39	600
6	667
× 100	65
600 ct.	17
	13
	4

sh.	ct.
19	32 r 17 ct. *Ans.*
18)347	93
18	500
167	593
162	54
5	53
× 100	36
500 ct.	17

Step 6. Increase divisors to 99 gradually. Note, as in the second example, the insertion of the 0 in the ct. answer.

sh.	ct.
10	41 r 13 ct. *Ans.*
39)406	12
39	1600
16	1612
× 100	156
1600 ct.	52
	39
	13

sh.	ct.
9	02 *Ans.*
72)649	44
648	100
1	144
× 100	144
100 ct.	

Give at least 10 sums like these.

LINEAR MEASURE

Aim

The four rules introducing chains and miles.

ADDITION

In Class 4 the main task is to extend the working of the four rules in measurement to three units instead of two. The first steps given for each rule are intended to take the place of revision, and they lead naturally into the new work to be attempted. The teacher should see that these early steps are done thoroughly, even though they may be done quickly. When giving the first examples to the class, check that they remember the rules for setting the sums down in their books and how to change yards into feet, feet into inches, etc.; and also when to put a 0 and when to leave an empty space.

Step 1. Addition, without carrying, of (*a*) 2 and (*b*) 3 items.

Demonstrate each type and then make at least ten examples of each for the class to do by themselves. Be sure that none of your own examples involve a carrying figure.

(*a*)

yd.	ft.	in.
2	0	5
+3	2	4

(*b*)

yd.	ft.	in.
2	0	3
1	1	2
+	1	5

yd.	ft.	in.
1	2	5
+2	0	6

yd.	ft.	in.
1	0	3
		2
+	1	4

yd.	ft.	in.
1	0	4
+	2	5

In the second example of part (*b*) there is a danger that one or two pupils ' lose ' the 1 yd. in the top line. Look out for this.

Step 2a. Addition, carrying from inches to feet.

Demonstrate one of each type of sum given, revising the ' underline ' working as you go. See that the backward or lazy child can deal with the ' zero feet everywhere ' in example (*c*).

```
      yd.  ft.    in.         yd.  ft.  in.          yd.  ft.  in.
(a)    3    0      9   (b)     4    0    4   (c)      4    0   11
      +4    1     11                1   11                     11
     ------------------       +5    0    3           +5    0    6
       7    2      8 Ans.    ---------------        ---------------
     ------------------      ---------------        ---------------
            1  12)20
            1      1 r 8
            -
            2
```

Step 2b. Addition, carrying from feet to yards.

The class should now be able to add three items straight away ; but when making your examples remember the slower children who may need to start with two items only. Vary your sums by leaving the empty space in the yards column in different items—sometimes the first, at others the second line, etc.

```
     yd.   ft.   in.
     14     2     4
      2     1     2
  +         2     3
  ---------------
     17     2     9  Ans.
  ---------------
      1   3)5
            1 r 2
```

When these preliminary steps are well understood the class may then go on to :

Step 3. Addition, carrying throughout.

Very little demonstration should be needed. When you make your own examples—the class should do by themselves not less

than 10—*be sure that the total is never more than* 21 *yd.* 2 *ft.* 11 *in.* (See next section.)

yd.	ft.	in.	yd.	ft.	in.	yd.	ft.	in.
9	1	3	8	2	11		2	9
2	2	5		1	6	1	2	6
+1	1	9	+4	2	4	+4	1	3

yd.	ft.	in.	yd.	ft.	in.	yd.	ft.	in.
2	1	9		1	9	6	2	4
1	2	6	4	2	11			11
+	2	11	+	1	2	+2	2	9

INTRODUCTION OF CHAINS

The next measure should be introduced now. First, make a chain measure out of rope or fibre, and have it ready for your first lesson. Tell the class:

'We have now learned to add yards, feet and inches together. If you look back at the sums you have just been doing you will see that the answer was never as much as 22 yards. This is because when we have 22 yards we often put them together and call them 1 chain.' (Write on blackboard '22 yards = 1 chain'.) 'We usually write "ch." for chain.' (Write '1 ch.' under the '1 chain'.) 'This measurement is very important because it is used a lot when making maps, measuring roads and plots, the bends in main roads and railway lines, and many other large things. It is the most important measure of big things next to a mile. Let us go out and measure some things which are a chain in length.' Now take your class outside and measure some things with your chain measure. You might spend one or two periods measuring and recording the length of the compound, the length of the football or netball pitch, etc. (See the notes for Class 2.) Make the records in ch. yd. ft. While this work is in progress tell the class that as a rule, when a thing is measured in ch., we ignore any odd inches, and say that the object is so many ch. yd. and ft. in length.

After the practical work, say: 'The first thing we have to be sure about before we can add together measurements in ch. yd. and ft. is the way of changing yards into chains. How many yards are there in a chain? (22). What do you think we ought to do?' (If you refer back to previous work the class will be able to tell you that we divide the yards by 22.)

Now demonstrate one after the other the following three sums. Explain that since dividing by 22 means long division, we must put our total of yards *two* lines below the answer line, with a line left empty for the answer to the long division. Use class co-operation as much as possible.

(i)

	ch.	yd.	
		19	
+		21	
	1	18	*Ans.*

$$\begin{array}{r} 1 \text{ r } 18 \\ 22\overline{)40} \end{array}$$

(ii)

	ch.	yd.
		12
+		10

(iii)

	ch.	yd.
		20
		19
+		16

Now make 10 examples for the class to do.

Step 1. Addition of yards and feet with chains in the answer and no carrying from feet to yards. Demonstrate the following and then let the children do 6 examples of 2, 3, and 4 items mixed.

	ch.	yd.	ft.
		18	0
+		21	2

	ch.	yd.	ft.	
		17	0	
		21	1	
		12	0	
+		18	1	
	3	2	2	*Ans.*

$$\begin{array}{r} 3 \text{ r } 2 \\ 22\overline{)68} \\ \underline{66} \\ 2 \end{array}$$

Step 2. Repetition of Step 1 together with the introduction of carrying from feet to yards. In this case the third line of the

underline working must be left blank and the total inserted in the fourth line under the yards column.

Demonstrate as follows:

ch.	yd.	ft.	
	14	2	
	12	1	
+	16	2	
1	21	2	*Ans.*

```
      1   3)5
          ---
     42    1 r 2
     --
      1 r 21
 22)43
    22
    --
    21
```

The teacher should now set 6 or more examples of this type for the children to do, with a maximum of 4 items.

Step 3. Addition of chains, yards and feet.

Demonstrate, stressing the rules for underline working.

ch.	yd.	ft.
19	16	2
	14	1
+ 5	21	2

Now make at least 10 examples for the class to do. *Remember these rules when doing so:*

1. Never more than four items to be added.
2. Never more than 2 ft. in ft. column.
3. Never more than 21 yd. in yd. column.
4. Never more than 79 ch. 21 yd. 2 ft. in answer.

INTRODUCTION OF MILES

When the class has thoroughly practised the addition of chains, etc., draw their attention (as in the introduction of chains) to the fact that their sums have never totalled as much as 80 chains.

Say 'This is because when we have 80 chains we put them together and call them 1 mile'. (Here revise the table of length by questions, and build up a blackboard table.)

12 in. = 1 ft.	22 yd. = 1 ch.
3 ft. = 1 yd.	80 ch. = 1 ml. (mile)

Refer back to the introduction of chains and obtain from the class the method of reducing (changing) chains into miles: then demonstrate and practise this step. It is obviously not practicable to do much measurement of miles but you should again point out that, as a rule, when we measure something in miles we do not include any odd feet, but measure in ml., ch., yd. only or perhaps even in miles and chains only.

Step 1. Addition of chains with miles and chains in the answer.

	ml.	ch.
		48
+		72
	1	40

	ml.	ch.
		65
+		73

	ml.	ch.
		58
		39
+		72

$$\begin{array}{r} 1\ \text{r}\ 40 \\ 80\overline{)120} \\ 80 \\ \hline 40 \end{array}$$

When demonstrating, stress the method of lay-out below the line, with the empty space left for the long division answer. Give the class about 20 examples to do.

Step 2. Addition of ch. and yd. with miles in answer, carrying ch. only.

	ml.	ch.	yd
		72	5
		64	8
+		68	3

Make 10 examples for the class to do. Be sure there is no carrying from yards to chains.

Step 3. Addition of ch. and yd., carrying throughout, with ml. in answer.

	ml.	ch.	yd.
		62	9
		10	8
+		34	11

Step 4. Addition of ml. ch. and yd., carrying throughout.

ml.	ch.	yd.	
	63	18	
5	0	14	
+23	78	0	
29	62	10	*Ans.*
1	1	* 1 r 10	

$$141 \qquad * \; 80\overline{)142} = 1 \text{ r } 62 \qquad \begin{array}{r} 142 \\ 80 \\ \hline 62 \end{array}$$

$$22\overline{)32} = 1 \text{ r } 10 \qquad \begin{array}{r} 32 \\ 22 \\ \hline 10 \end{array}$$

* Stress the need for leaving this space empty for the long division answer.

Make at least 10 examples for the class to do.

SUBTRACTION

Step 1. Subtraction of yd. ft. in., without carrying.

Demonstrate one sum and give the class ten examples to do by themselves.

yd.	ft.	in.
15	2	11
− 9	0	8

yd.	ft.	in.
12	1	9
− 8	0	8

yd.	ft.	in.
14	2	10
− 3	2	9

yd.	ft.	in.
19	0	10
− 9	0	2

Step 2. Subtraction, carrying from yd. to ft. *No* insertions except *for decomposed figure.*

$$\begin{array}{rrrl} \text{yd.} & \text{ft.} & \text{in.} & \\ 7 & & & \\ \cancel{8} & 0 & 8 & \\ -4 & 2 & 3 & \\ \hline 3 & 1 & 5 & \textit{Ans.} \\ \hline \end{array} \qquad \begin{array}{rrr} \text{yd.} & \text{ft.} & \text{in.} \\ 8 & & \\ \cancel{9} & 1 & 9 \\ -2 & 2 & 2 \\ \hline & & \\ \hline \end{array}$$

Say: '8 in., take away 3 in., leaves 5 in.' (Write 5 in.) '0 ft., take away 2 ft., I cannot, so I take 1 yd. from my 8 yd., leaving 7'—(cross out 8, write 7 on the right)—'and change the yard into feet. 3 ft., take away 2 ft., leaves 1 ft.' (Write 1 ft.) '7 yd., take away 4 yd., leaves 3 yd.' (Write 3 yd.) 'Answer, 3 yd. 1 ft. 5 in.'

At least 10 examples should be worked by the class. When making them up, the rule is 'Yards and inches bigger in top line, feet bigger in bottom line'.

Step 3. Subtraction, carrying ft. to in. only.

Demonstrate one or two sums of this type. Here is an example:

$$\begin{array}{rrrl} \text{yd.} & \text{ft.} & \text{in.} & \\ & 1 & & \\ 12 & \cancel{2} & 9 & \\ -10 & 0 & 11 & \\ \hline 2 & 1 & 10 & \textit{Ans.} \\ \hline \end{array}$$

Say: '9 in., take away 11 in., I cannot. So I take 1 ft. from my 2 ft., leaving 1 ft.'—(cross out the 2, write 1 on the right)—'and change the 1 ft. to inches. 1 ft. equals 12 in.; 12 and 9 equal 21 in. 21 in., take away 11 in., leaves 10 in.' (Write 10 in.) '1 ft., take away 0 ft., leaves 1 ft.' (Write 1 ft.) '12 yd., take away 10 yd., leaves 2 yd. Answer, 2 yd. 1 ft. 10 in.'

Now make at least 10 examples for the class to do. The rule is 'yards and feet bigger in the top line, inches bigger in the bottom line'.

Step 4. Subtraction, carrying throughout.

Demonstrate, using explanations given in Steps 2 and 3.

yd.	ft.	in.
18		
~~19~~	~~1~~	4
− 8	2	6

When setting your own examples for the class to do, great care must be taken not to have a 0 in the feet column of the top line at this stage. This is a separate difficulty which is dealt with in the next step—5. Set at least 10 for the class, using the following as models:

yd.	ft.	in.	yd.	ft.	in.	yd.	ft.	in.
9	2	0	8	1	9	15	2	6
−4	2	3	−6	1	10	− 5	2	9

Step 5. Subtraction, carrying through 0 ft. in top line.

Pay attention to the crossing out of decomposed figures and note that the yard brought to the feet column has to be written in as 3 ft.

yd.	ft.	in.
17	~~3~~ 2	
~~18~~	~~0~~	6
− 5	2	11

When demonstrating the example, say: '6 in., take away 11 in., I cannot. So I must take a foot from the feet column. But there are no feet in the feet column. What can I do?' (If the class can give the answer, so much the better.) 'I take a yard from my yards column, cross out the 18 and leave 17 yd.; I change the yard to feet, making 3 ft. I cross out the 0 in the feet column and write 3 on the left of it. Now I can take one of those 3 ft., leaving 2'—(cross out the 3 and insert the 2 on the right of it)—'and change the 1 ft. to 12 in. 12 and 6 is 18 in. 18, take away 11, leaves 7 in. Write the 7 . . .', and continue as in Steps 2 and 3.

More than one example may be needed in your demonstration,

as this step is not easy. You must give the class at least 10 examples to do on their own. The rules for making the examples are: 'Yards greater in top line: 0 feet in top line: inches smaller in top line.'

yd.	ft.	in.	yd.	ft.	in.	yd.	ft.	in.
15	0	4	18	0	2	12	0	3
− 6	1	9	−12	2	4	− 1	1	11

Before going on to Step 6 give a period of revision *practice* on Steps 1–5.

Step 6. Chains introduced—subtraction of ch. yd. ft., carrying ch. to yd. only.

ch.	yd.	ft.
18	40	
~~19~~	~~18~~	2
−18	19	0

Allow the class to use the helping figure in the yards column as well as for the decomposed figure of chains. Any pupil able to do so may be allowed to dispense with either or both figures; the addition of the 22 yards carried to the yards in the yards column must be done mentally. Do *not* therefore do it like this on the blackboard when demonstrating:

$$\begin{array}{r} 18 \\ +22 \\ \hline \end{array}$$

Use the same form of explanation as that given in Steps 2 and 3, and set at least ten examples for the class to do; the rules are—'chains and feet greater, yards smaller in top line'.

Step 7. Subtraction of ch. yd. ft., carrying throughout.

ch.	yd.	ft.
28	36 ~~14~~	
~~29~~	~~15~~	1
−17	18	2

Demonstrate and explain as before. Do not allow any helping figure in the ft. column. Note position of second helping figure—

22 + 14 yd.—in yard column. Give 20 examples for the class, the rules for making them being—chains greater, yards and feet smaller, in top line than in bottom line.

Step 8. Subtraction of ch. yd. ft., carrying through 0 yd.

ch.	yd.	ft.
11	~~22~~ 21	
~~12~~	~~0~~	1
− 9	18	2

Follow the explanation given in Step 5. Then make 20 examples for the class to do. The rules are: 'In top line, ch. greater, yd. 0, ft. smaller than in bottom line'.

Give a period of *revision-practice* on Steps 6–8 before going on.

Step 9. Miles introduced—subtraction of ml. ch. yd., carrying ml. to ch. only.

ml.	ch.	yd.
42	142	
~~43~~	~~62~~	18
− 18	71	16

Demonstrate and explain as in Steps 2, 3 and 6. Allow any necessary helping figure in all columns. Encourage pupils to dispense with them as soon as they feel able to do so. Make 10 examples for the class to do. The rules are: 'ml. and yd. greater, ch. smaller in top line than in bottom line'.

Step 10. Subtraction of ml. ch. yd., carrying throughout.

ml.	ch.	yd.
47	133 ~~53~~	38
~~48~~	~~54~~	~~16~~
− 21	62	19

Demonstrate and explain as before. Make at least 10 examples for the class to do. The rules for the top line are: 'ml. greater, ch. and yd. smaller than in bottom line'.

Step 11. Subtraction, carrying across 0 ch.

ml.	ch.	yd.
18	~~80~~ 79	32
~~19~~	~~0~~	~~10~~
−12	27	13

Demonstrate as in Steps 5 and 8. Give at least 10 sums for the class to do by themselves. The rules for making them are: 'In the top line, ml. greater, ch. 0, yd. smaller than in bottom line'.

Give at least one period of *revision-practice* on Steps 9–11 and one on all subtractions.

SHORT MULTIPLICATION

Aim

Short multiplication of yards, feet and inches.

Step 1. Revision.

Give one demonstration example of multiplication of yards and feet and one of multiplication of feet and inches from Class 3, Step 2, in each case. Then let the class work two or three sums of each type in their books. This step should be covered quickly.

Step 2. Short multiplication of yd. ft. and in., with answer always less than 22 yd. Demonstrate these two examples with the help of the class:

(*a*)

yd.	ft.	in.	
2	2	11	
		×7	
20	2	5	*Ans.*
6	6	12)77	
14	14	6 r 5	
20	3)20		
	6 r 2		

(*b*)

yd.	ft.	in.	
4	0	9	
		×4	
17	0	0	*Ans.*
1	3)3	12)36	
16	1 r 0	3 r 0	

The examples which can be worked are limited. Here are ten : you should make ten more yourself for the class to do.

	yd.	ft.	in.		yd.	ft.	in.
(i)	3	0	10	(ii)	2	0	9
			×6				×7

	yd.	ft.	in.		yd.	ft.	in.
(iii)	3	2	9	(iv)	8	1	10
			×5				×2

	yd.	ft.	in.		yd.	ft.	in.
(v)	4	2	11	(vi)	3	2	4
			×4				×5

	yd.	ft.	in.		yd.	ft.	in.
(vii)	1	2	11	(viii)	6	0	5
			×11				×3

	yd.	ft.	in.		yd.	ft.	in.
(ix)	7	0	4	(x)	5	0	4
			×3				×4

Step 3. Short multiplication of yd. and ft. with ch. in answer only

In your blackboard demonstration stress the need to leave a space for the 'long division answer' in the underline working in the yards column. This space is marked with a * in the example below :

ch.	yd.	ft.
	18	2
		×7
5	20	2 *Ans.*
5	4	3)14
	126	4 r 2
	* 5 r 20	
	22)130	
	110	
	20	

Set at least 10 examples for the class to do.

LONG MULTIPLICATION

Aim

Long multiplication of yards, feet and inches.

Step 1. Long multiplication of yd. ft. and in. with a single yard in the multiplicand.

It should be noted by the teacher that this is only a transitional stage before the children move on to work with chains, and although the number of examples available with answers up to 21 yd. 2 ft. is limited this does not matter.

Demonstrate, explaining as in Step 1 :

yd.	ft.	in.	
1	0	6	
		× 15	
17	1	6	*Ans.*
2	3)7	12)90	
	2 r 1	7 r 6	

Give the class 10 examples of this step to do.

Step 2. Long multiplication of yd. and ft. converting to ch. in answer.

Demonstrate, explaining each point as in Step 1 above :

	ch.	yd.	ft.	
(*a*)		9	2	
			× 17	
	7	10	1	*Ans.*
	7	11	3)34	
		*153	11 r 1	
		7 r 10		
		22)164		
		154		
		10		

* Multiply by the single digit figure in the top line to get this answer.

	ch.	yd.	ft.
(*b*)		13	2
			×15
	9	7	0
	9	10	3)30
		65	10 r 0
		130	
		9 r 7	
		22)205	
		198	
		7	

Now make at least 10 examples for the class to do, with types (*a*) and (*b*) mixed. Keep your multiplier fairly low—say below 60.

SHORT DIVISION

Aim

Short division of yards, feet and inches.

The class should have learned all the processes needed for the first two steps at least in the previous year. Work all demonstrations, therefore, with class co-operation.

Step 1. Short division of yd. ft. in., carrying yd. to ft. only, *no* remainders, *no* helping figures.

Using the form of explanation given in 'Division of Yd. and Ft.', Class 3, Step 2, but omitting the writing of any helping figure, demonstrate not more than two of the examples below. Then give the remainder to the class to do by themselves.

	yd.	ft.	in.		yd.	ft.	in.
8)	18	2	8	9)	21	0	9
4)	21	1	8	5)	16	2	10
6)	16	0	6	7)	9	1	7

It should be possible in the same period to go on to

Step 2. Short division, carrying from ft. to in. only.

All answers in this section will have 0 ft. Demonstrate and practice as in Step 1.

	yd.	ft.	in.		yd.	ft.	in.		yd.	ft.	in.
9)	18	2	3	7)	21	1	2	5)	20	2	1
11)	11	1	10	10)	20	2	6	6)	18	2	6

Step 3. Short division of yd. ft. in., carrying throughout, no remainders.

Demonstrate one example of each section (*a*) and (*b*) and (*c*), then give the rest to the class to do on their own.

(*a*) Answers with quantities in all columns.

	yd.	ft.	in.		yd.	ft.	in.		yd.	ft.	in.
7)	18	1	5	4)	13	2	8	10)	16	1	2
5)	19	1	4	11)	21	1	2	6)	15	2	6
9)	21	2	3	6)	20	1	6	8)	14	2	8
				11)	17	1	3				

(*b*) Answers with 0 ft.

	yd.	ft.	in.		yd.	ft.	in.		yd.	ft.	in.
9)	20	2	3	5)	21	1	7	11)	13	2	3
8)	17	2	4	7)	15	2	3				

(*c*) Answers with 0 yd. (blank space).

	yd.	ft.	in.		yd.	ft.	in.		yd.	ft.	in.
9)	8	2	3	11)	9	1	5	10)	4	1	4
8)	7	2	4	6)	2	2	6				

Step 4. Short division of yd. ft. in. with remainders.

Demonstrate and practice as before in Step 3. It may be necessary for you to make more examples of each section for the class to do.

(*a*) Answers with a quantity in the inches column.

	yd.	ft.	in.
12)	19	2	11
9)	16	1	9
8)	11	2	6
11)	15	1	7
7)	18	1	0

(*b*) Answers with 0 in inches column. (This only happens when there is no carrying from feet to inches and the inches figure in the dividend is smaller than the divisor.)

	yd.	ft.	in.
8)	21	1	4
12)	16	0	9
11)	18	1	6
5)	18	1	2
9)	21	0	5

LONG DIVISION OF LENGTH

Aim

Long division of yd. ft. in.

Introduction

Revise with the class the method of long division—demonstrate on the blackboard one or two sums from the later steps of Long Division of Number, and let the class spend the remainder of the period doing examples of this work. Make sure that they have a good knowledge of Long Division method before going on.

Step 1. Long division of yd. ft., with blank space (0 yd.) in answer.

Say: 'Now we are going to use the long division method for dividing lengths. We shall begin with yards and feet. This is

how we must put our sums down.' (Use blackboard as you explain.)

'First I write - - - - - -	yd.	ft.
'Then I leave a line for the answer - -		2
'Then I write my sum - - - -	13)8	2
	×3	24
	24 ft.	26
		26

You can explain the sum like this:

'13 into 8 yd. will not go. So I have no yards in the answer. Do I put a 0 in the yard place of the answer? . . Why not? (Because there is no figure on the *left* of the yard column.)

'Now I must change my yards into feet. How do I do it? Where do I do it?' (Remind the class that all changing is done in the *higher place*. Yards to feet in the yard column, feet to inches in the feet column. They may remember what they were taught in Long Division of Money, Step 3.)

'3 times 8 are 24. 24 what? . . . Write the word against the figure. Bring 24 ft. to the feet column. Add them to the feet already there. 24 and 2 make 26.

'13 into 26 ft. goes . . . twice. Write 2 in the feet place of the answer line. Twice 13 are 26. Write 26 under the 26 ft. 26 from 26 leaves nothing. Answer 2 ft.'

Now give the class at least 10 sums to work by themselves.

Step 2. Division of yd. ft. in., 0 yd. in answer line.

Demonstrate and explain as in Step 1, extending your explanation to the inches column.

yd.	ft.	in.
	2	8
14)12	1	4
×3	36	108
36 ft.	37	112
	28	112
	9	
	×12	
	108 in.	

Make up 10 examples for the children to do themselves.

Step 3. Division of yd. ft. and in. with:

(*a*) Quantities in all places of answer.

(*b*) 0 feet in answer.

(*c*) 0 inches in answer.

(*a*)

	yd.	ft.	in.	
	1	1	4	*Ans.*
13)	18	2	4	
	13	15	48	
	5	17	52	
	×3	13	52	
	15 ft.	4		
		×12		
		48 in.		

(*b*)

	yd.	ft.	in.	
	1	0	11	*Ans.*
13)	16	2	11	
	13	9	132	
	3	11	143	
	×3	×12	143	
	9 ft.	132 in.		

(*c*)

	yd.	ft.	in.	
	1	1	0	*Ans.*
16)	21	1	0	
	16	15		
	5	16		
	×3	16		
	15 ft.			

Set 5 examples of each of the three types for the class to do themselves.

Step 4. Long division of ch. yd. and ft.

Point out to the class that the chain measure is most important

in outdoor work, as they learned earlier in the year. In arithmetic we only use it commonly when we are working sums including miles, such as they will see in Class 5. We are preparing for the work of Class 5 now by learning how to divide sums in which there are chains. Remind them of what was said about leaving out inches when working with chains (Introduction of Chains, page 109).

The method of working is the same as for yards, feet and inches. The only difference is in the multiplier used to change chains into yards. Write the sum on the blackboard :

	ch.	yd.	ft.	
	4	7	1	*Ans.*
17)	73	14	2	
	68	110	15	
	5	124	17	
	×22	119	17	
	110 yd.	5		
		×3		
		15 ft.		

Demonstrate another sum of the same type and then set at least 10 for the class to do. *Rules:* 'Chains below 80 in dividend, divisor always smaller than the number of chains in dividend, and yards in answer not to exceed 9'.

Step 5. Long division of ch. yd. ft. with two operations of division in yd. column.

The class has already met the two-stage division in money and possibly in the inches column of mensuration sums. It is treated as a special topic here because the final remainder in the yards column (if any) has to be changed into feet. The class must therefore have practice at sums where there are two stages, so that they do not change the first remainder into feet.

Demonstrate this type of sum. *Do not* draw attention to the difficulty, but simply do the division.

		ch.	yd.	ft.	
(*a*)		4	15	2	*Ans.*
	16)	75	8	2	
		64	242	30	
		11	250	32	
		×22	16	32	
		22	90		
		220	80		
		242 yd.	10		
			×3		
			30 ft.		

At least five examples of this type should be set for the children themselves to do.

(*b*) At this point the additional complication of 10 yd. or 20 yd. in the answer (i.e. two processes of division of yds. with 2nd answer 0) should be dealt with. The following example should be worked on the blackboard :

	ch.	yd.	ft.	
	2	10	2	*Ans.*
27)	67	2	0	
	54	286	54	
	13	288	54	
	×22	27	54	
	26	18 *		
	260	×3		
	286 yd.	54 ft.		

* Say : '27 into 18 will not go, put 0 in the units place of the yards column of the answer. Change 18 yd. to feet,' etc.

At least 6 examples like this should be set for the children to do.

Step 6. Miscellaneous zero difficulties.

(*a*) The medial zero (i.e. with or without carrying from ch. to yd., but with no yd. in answer). Demonstrate as follows, making sure that the children understand each step in the working :

(i)

```
       ch.     yd.     ft.
        3       0       2  Ans.
     ______________________
    22)66      14       2
       66      ×3      42
       --      --      --
             42 ft.    44
                       44
                       --
```

(ii)

```
       ch.     yd.     ft.
        2       0       2  Ans.
     ______________________
    37)75       2       2
       74      22      72
       --      --      --
        1      24      74
     ×22       ×3      74
     ---       --      --
     22 yd.  72 ft.
```

(*b*) Dividend with 0 yd.

```
       ch.     yd.     ft.
        2       3       1  Ans.
     ______________________
    20)43       0       2
       40      66      18
       --      --      --
        3      66      20
     ×22       60      20
     ---       --      --
     66 yd.     6
               ×3
               --
             18 ft.
```

(*c*) Answer with 0 ch.

	ch.	yd.	ft.	
	*	14	2	*Ans.*
28)	18	14	2	
	×22 *	396	54	
	36	410	56	
	360	28	56	
	396 yd.	130		
		112		
		18		
		×3		
		54 ft.		

* Revise rule for leaving empty space. Say '28 into 18 will not go. Leave the ch. place empty in answer line. Change 18 ch. to yd.', etc.

(*d*) Answer with 0 ch. and 0 yd.

	ch.	yd.	ft.	
	*	*	2	*Ans.*
68)	2	1	1	
	×22 *	44	135	
	44 yd.	45	136	
		×3 *	136	
		135 ft.		

* The form of explanation is given against example (*c*) above. Points to watch are marked *.

The zero difficulties detailed in (*a*) to (*d*) above are classified here so that the teacher will not necessarily expect the children to work them without any preparation or advice. Each of the points is simple, however, and the majority of the children should grasp them quickly with little difficulty.

At least 20 examples of these various types mixed should now be set for the children to do. The examples of types *a*(ii) and (*d*)

are limited and 3 or 4 of each type is sufficient ; but more practice should be given with type *a*(i), (*b*), and (*c*).

Step 7. Division with remainders.

There is little difficulty with the arithmetic of this step, but the class must be sure what the remainder is. In every case the remainder must be shown as so many in., or ft., etc.

(*a*) With quantity in inches column and remainder in inches.

```
       yd.      ft.      in.
        1        1        2 r 3 in.  Ans.
    15)20        2        9
       15       15       24
       --       --       --
        5       17       33
       ×3       15       30
      ---       --       --
       15 ft.    2        3 *
               ×12
               ---
                24 in.
```

* Say ' 3 in. remainder. There is nothing to bring down so I cannot divide further '. Write the remainder in answer line : ' r 3 in. ' Stress the word ' inches ' and underline it on the blackboard.

(*b*) With quantity in inches column and remainder in feet (or feet and inches).

```
       yd.      ft.      in.
        1        0        5 r 1 ft.  Ans.
    19)21        2       11
       19        6       96
       --        -      ---
        2        8      107
       ×3      ×12       95
      ---      ---       --
        6 ft.   96 in.   12 *
```

* Say ' 19 into 12 will not go, and there is nothing else to bring down, so this is the remainder. But we never write 12 in. but always 1 ft. ', and then write ' r 1 ft. '.

(*c*) With last quantity in feet column and remainder in feet.

	ch.	yd.	ft.
	2	7	2 r 1 ft. *Ans.*
29)	68	2	2
	58	220	57
	10	222	59
	×22	203	58
	220 yd.	19	1 *
		×3	
		57 ft.	

* Explain in a similar way as for inches in (*a*) above.

(*d*) With last quantity in feet column and yd. and ft. in remainder.

	ch.	yd.	ft.
	1	16	2 r 6 yd. 2 ft. *Ans.*
43)	75	19	1
	43	704	105
	32	723	106
	×22	43	86
	64	293	3)20 *
	640	258	6 yd. 2 ft.
	704 yd.	35	
		×3	
		105 ft.	

* Proceed on the same lines as in (*b*) above, but teach the class to reconvert the feet and write down the equivalent in yd. and ft. as shown here.

The class should do about 20 examples covering the various types (*a*) to (*d*) dealt with above.

You should finish this topic by giving one or two periods of *revision-practice* on long division, following this with at *least one additional period* devoted to practice on the whole topic of length in the four rules.

CAPACITY

There is considerable doubt about the position of the quart in East Africa. Its current use is limited, but it is included here for the sake of completeness.

The teaching of capacity in Class 4 is restricted to the teaching of this measure and its relation to the other measures of capacity already considered in Class 3.

Aim

To introduce quarts in the 4 rules with debes, gallons and pints.

Introduction

Apparatus—At least 2 pint mugs, 4 quart containers (preferably a commercial one like a quart bottle or tin), and a gallon tin.

The children have already discovered by practical work in measuring capacities in Class 3 that:

8 pints = 1 gallon
and 4 gallons = 1 debe

In a similar way they should now see that:

2 pints = 1 quart
and 4 quarts = 1 gallon

thus extending the table already learnt.

ADDITION

Step 1. Introduce in 2 quantities only, the carrying of quarts to gallons as follows:

	gall.	qt.	pt.	
		2	1	
+		3	0	
	1	1	1	*Ans.*
	1	4)5 = 1 r 1		

	gall.	qt.	pt.	
		3	1	
+		3	0	
	1	2	1	*Ans.*
		4)6 = 1 r 2		

Children to do 5 or 6 examples quickly.

Step 2. Carrying from both quantities, qt. and pt.

	gall.	qt.	pt.	
		2	1	
+		2	1	
	1	1	0	*Ans.*
		1	2)2	
		4	1 r 0	
		4)5		
		1 r 1		

	gall.	qt.	pt.	
		3	1	
+		2	1	
	1	2	0	*Ans.*
		1	2)2	
		5	1 r 0	
		4)6		
		1 r 2		

Children to do 5 or 6 more examples of this type quickly.

Step 3. Addition of two quantities, gallons and quarts, with carrying to debes (with or without carrying from quarts).

	de.	gall.	qt.	
		1	2	
+		2	3	
	1	0	1	*Ans.*
		1	4)5	
		3	1 r 1	
		4)4		
		1 r 0		

	de.	gall.	qt.	
		3	3	
+		1	3	
	1	1	2	*Ans.*
		1	4)6	
		4	1 r 2	
		4)5		
		1 r 1		

Children to do 5 of this type quickly.

Step 4. Introduce 3 quantities, first with two items, then with 3 and 4.

(i)

de.	gall.	qt.
1	2	1
+2	2	1

de.	gall.	qt.
3	3	0
+1	3	1

(ii)

de.	gall.	qt.
2	0	1
3	3	1
+3	2	1

de.	gall.	qt.
3	2	0
4	2	2
3	3	1
+10	3	3

The children should then work 10 examples set by the teacher on these lines.

The teacher should note that the steps given here are very short. This is a small topic which should be dealt with fairly briskly, and the demonstration, together with all of the 4 steps indicated above, should be completed in 3 or at the very most 4 lessons. The same procedure applies in the other rules which follow.

SUBTRACTION

Step 1. Subtraction of pt., and qt. and pt., from qt. and pt., with carrying figure.

qt.	pt.	qt.	pt.	qt.	pt.	etc.
1	0	3	0	3	0	
–	1	–	1	–1	1	

Children should do at least 10 of these mentally, writing answers only down in their books.

Step 2. Subtraction of (i) qt. and pt., then (ii) gall. qt. and pt. from gall. qt. and pt.

	gall.	qt.	pt.		gall.	qt.	pt.
(i)	3	1	1	(ii)	3	2	0
	–	3	0		–1	3	1

Children should do at least 5 of each type themselves.

Step 3. Subtraction of (i) gall. and qt., then (ii) de. gall. and qt. from de. gall. and qt.

	de.	gall.	qt.		de.	gall.	qt.
(i)	3	2	2	(ii)	7	1	0
	–	3	3		–2	3	2

Again the children should themselves do at least 5 of each type which the teacher should set for them.

MULTIPLICATION

Step 1. Short multiplication.

While this class is capable of understanding and dealing with 3 quantities straight away, it is advisable in the first blackboard demonstration to multiply gallons and quarts only, giving de.

gall. and qt. in the answer, before proceeding to 3 quantities in the sum itself, as follows:

	de.	gall.	qt.	
(*a*)		3	2	
			×3	
	2	2	2	*Ans.*
		1	4)6	
		9	1 r 2	
		4)10		
		2 r 2		

	de.	gall.	qt.	
(*b*)	2	2	3	
			×4	
	10	3	0	*Ans.*
	2	3	4)12	
	8	8	3 r 0	
	10	4)11		
		2 r 3		

Set 10 examples for the children to do, mainly of type (*b*) but with two or three of (*a*) included.

Step 2. Long multiplication.

This has already been taught in careful steps both in money and in linear measurement, so it is not necessary to grade the long multiplication of capacity so finely, and it can well be taught in the same brief stages as short multiplication above.

	de.	gall.	qt.	
(*a*)		2	1	
			×13	
	7	1	1	*Ans.*
		3	4)13	
		26	3 r 1	
		4)29		
		7 r 1		

	de.	gall.	qt.	
(*b*)	2	3	2	
			×17	
	48	3	2	*Ans.*
	14	8	4)34	
	34	51	8 r 2	
	48	4)59		
		14 r 3		

The children should then do 10 or more examples of types (*a*) and (*b*) mixed.

DIVISION

Step 1. Short division of gallons and quarts only. The teacher demonstrates the first one and the children then work the rest themselves :

gall.	qt.	gall.	qt.	gall.	qt.
5) 3	3	2) 3	2	3) 2	1
7) 3	2	6) 3	0	2) 3	2
3) 1	2	4) 2	0	5) 2	2
4) 3	0	7) 1	3	2) 3	0

Step 2. Short division of debes, gallons and quarts.

de.	gall.	qt.
4)7	2	0
5)8	0	2
7)46	1	2

Children to work at least 10 more examples set by teacher.

Step 3. Long division of de. gall. and qt.

	de.	gall.	qt.	
	3	3	2	*Ans.*
13)	50	1	2	
	39	44	24	
	11	45	26	
	×4	39	26	
	44 gal.	6	..	
		×4		
		24 qt.		

	de.	gall.	qt.	
	3	0	3	*Ans.*
29)	92	1	3	
	87	20	84	
	5	21	87	
	×4	×4	87	
	20 gall.	84 qt.	..	

The teacher should set at least 12 more examples for the children themselves to do.

WEIGHT

Aim

Completion of 4 rules in lb. oz.

The work to be done in Class 4 is limited by the fact that their knowledge of long division method only extends to divisors of two digits. Weight-units are therefore confined to lb. and oz., and special preparation is required for short division.

Preparation for Division and for Reduction

Before any mechanical work on weight is done in Class 4, the following work in preparation for division sums and division 'below the line' (i.e. reduction) should be introduced into the daily tables and mental drill.

First let the class multiply 16 by 2, 3, 4 and 5, and when they have obtained the answers, record the results in the following table form:

$$16 \text{ oz.} \times 1 = 16 \text{ oz.} = 1 \text{ lb.}$$
$$16 \text{ oz.} \times 2 = 32 \text{ oz.} = 2 \text{ lb.}$$
$$16 \text{ oz.} \times 3 = 48 \text{ oz.} = 3 \text{ lb.}$$
$$16 \text{ oz.} \times 4 = 64 \text{ oz.} = 4 \text{ lb.}$$
$$16 \text{ oz.} \times 5 = 80 \text{ oz.} = 5 \text{ lb.}$$
$$\text{etc. to } 16 \text{ oz.} \times 10 = 160 \text{ oz.} = 10 \text{ lb.}$$

This table must now be learned thoroughly. At the same time give mechanical short division of all the numbers within the range of that table, so that the children can divide any number between 16 and 160 by 16 and get the right answer straight away.

First work through division of numbers between 17 and 31. Then numbers between 32 and 47: then 48 and 63: then 64 and 79.

When each section has been done mechanically, introduce it into the daily mental work.

ADDITION

Step 1. (*a*) Addition of 2 items of ounces carrying to lb. in answer only.

	lb.	oz.	
		13	
+		15	
	1	12	*Ans.*
	1	16)28	
		1 r 12	

(*b*) Addition of oz. to lb. oz.

	lb.	oz.	
	9	12	
+		7	
	10	3	*Ans.*
	1	16)19	
		1 r 3	

	lb.	oz.	
		14	
+	19	11	
	20	9	*Ans.*
	1	16)25	
		1 r 9	

(*c*) Addition of lb. oz. to lb. oz.

lb.	oz.	
15	10	
+19	10	
35	4	*Ans.*
1	16)20	
	1 r 4	

This step presents no special difficulty—or should not—to Class 4, and two periods should be enough to establish setting down and method. Make about 15 examples—5 of each.

Step 2. Addition of 3 and 4 items. (Make 10 examples of each type.)

lb.	oz.	
19	10	
4	15	
+21	6	
45	15	*Ans.*
1	16)31	
	1 r 15	

lb.	oz.	
27	4	
16	12	
32	13	
+11	9	
88	6	*Ans.*
2	16)38	
	2 r 6	

lb.	oz.	
14	11	
	15	
28	9	
+	5	
44	8	*Ans.*
2	16)40	
	2 r 8	

SUBTRACTION

Step 1. Oz. from lb. oz., carrying.

Note on Helping Figures:

Helping figures should always be allowed for the decomposed figure of lb. In the early stages they may be allowed in the oz. column also: for many children this will be essential: but the teacher should make his aim the gradual disappearance of this second helping figure, so that by the end of the topic only the decomposed figure is written anew. Really able children may be permitted at the teacher's discretion to dispense with *all* helping figures as soon as they have the principle of the sum by heart.

After demonstration, let the class do first at least 10 examples like this :

$$\begin{array}{rr}
\text{lb.} & \text{oz.} \\
13 & 25 \\
\cancel{14} & \cancel{9} \\
- & 12 \\
\hline
13 & 13 \ \textit{Ans.} \\
\hline
\end{array}$$

Step 2. Lb. oz. from lb. oz.

After demonstration, the class should do about 10 examples.

$$\begin{array}{rr}
\text{lb.} & \text{oz.} \\
34 & \\
\cancel{35} & 11 \\
-27 & 15 \\
\hline
7 & 12 \ \textit{Ans.} \\
\hline
\end{array}$$

Forms of demonstration-explanation should be based on those given for the early steps of subtraction in linear measure.

SHORT MULTIPLICATION

Step 1. Multiplication of oz. with lb. in answer.

The changing of oz. to lb. in multiplication will always be done by the long division method because the number of oz. will often exceed 160.

$$\begin{array}{lr}
\text{lb.} & \text{oz.} \\
 & 15 \\
 & \times 8 \\
\hline
7 & 8 \ \textit{Ans.} \\
\hline
\end{array}
\qquad
\begin{array}{lr}
\text{lb.} & \text{oz.} \\
 & 9 \\
 & \times 12 \\
\hline
6 & 12 \ \textit{Ans.} \\
\hline
\end{array}$$

$$\begin{array}{r}
7 \\
16\overline{)120} \\
112 \\
\hline
8
\end{array}
\qquad
\begin{array}{r}
6 \\
16\overline{)108} \\
96 \\
\hline
12
\end{array}$$

Ten sums at least of this type should be given to establish method and accuracy.

Step 2. Multiplication of lb. oz.

lb.	oz.		lb.	oz.	
9	13		10	7	
	×11			×9	
107	15	*Ans.*	93	15	*Ans.*
8	8		3	3	
99	16)143		90	16)63	
	128			48	
	15			15	

At least 20 examples required for class practice.

DIVISION

Step 1. Division of lb. oz. without remainder. No carrying.

The step itself involves no difficulty.

lb.	oz.		lb.	oz.	
7)63	14		11)110	11	
9	2	*Ans.*	10	1	*Ans.*

Give not less than 10 examples for practice.

Step 2. Division without remainder, with carrying.

The carried lb. figure to be changed into oz. must never exceed 5. Allow helping figure in oz. column but encourage able children to dispense with it.

lb.	oz.		lb.	oz.	
9)84	~~6~~ 54		11)104	8 88	
9	6	*Ans.*	9	8	*Ans.*

Give not less than 10 examples for practice.

Step 3. Division, without remainder, with 0 lb. in answer.

Revise the question of leaving a blank space, as shown in Linear Measure, Class 3, Step 2, etc. Ten sums worked by the class should be sufficient to establish this step.

lb.	oz.		lb.	oz.	
9)5	~~1~~ 81		11)3	~~7~~ 55	
	9	*Ans.*		5	*Ans.*

Step 4. Division with remainders.

(*a*) Quantity in oz. column.

	lb.	oz.	
12)	85	~~11~~ 27	
	7	2 r 3 oz.*	*Ans.*

	lb.	oz.	
9)	102	~~14~~ 62	
	11	6 r 8 oz.*	*Ans.*

* Stress that the nature of remainder must be shown. See Division of Money, Length, etc., Class 3.

(*b*) 0 in oz. column.†

	lb.	oz.	
7)	105	4	
	15	0 r 4 oz.	*Ans.*

	lb.	oz.	
8)	104	7	
	13	0 r 7 oz.	*Ans.*

† In short division, this only happens when there is no carrying figure of lb.

For model forms of explanation to be used, refer to the appropriate sections of the manual for Linear Measure, Class 3.

PERIMETER

It is suggested that when the teacher is going to teach Perimeter, he makes no mention of the word. He will give his class a simple problem such as the following.

'Let us consider the putting up of a fence around our football field—you know how we sometimes lose the ball in the bush, it would be so convenient to have a fence. Who can estimate the length of the field? (this is good practice at all times); and the breadth?'

The teacher puts a sketch on the blackboard with L and B measurements now marked in. He points to the other length side and elicits it is the same; similarly with the other B side. He asks what is the total length of fencing required to go all round the field.

When he has received the answer, he explains that one word is used to mean 'distance all round' a square, rectangle or any sided figure—it is *perimeter*. So, when we find perimeter of the field, we find the total fencing required.

Point out that

In a square, perimeter is $4 \times L$;

In a rectangle, perimeter is $(2 \times L) + (2 \times B)$;

In any other shape, perimeter is the sum of all sides.

Give 10 sums for each.

MONEY

Aim

To teach the four rules in £ and sh.

Step 1. Show the class an E. African 20 shilling note, bringing attention to the inscription which reads 'Twenty Shillings or One Pound'. Give the class some preliminary mental work as follows :

(*a*) How many shillings in £4, £7, £6, £9, £11, etc.

(*b*) How many pounds in sh. 20, 60, 100, 180, 200, etc.

(*c*) How many pounds and shillings in sh. 27, 49, 75, 82, etc.

Give the symbol (£) which represents pounds.

Step 2. Teach the short method of dividing by 20.

Ask the class how many 20s in 84, and what is the remainder. This will be given.

Now ask how many 20s in 346, and the remainder. Some will probably fail, but even if not, it will be a delayed answer. Tell the class you will show them a quick method and refer back to the first sum. Proceed to the following demonstration of crossing off the 0 of the 20, and the last figure of the dividend which is the remainder of shillings, then dividing by 2.

$$2\cancel{0})\underline{8\cancel{4}}$$
$$4 \text{ r } 4$$

Now proceed to do the same with 346.

$$2\cancel{0})\underline{34\cancel{6}}$$
$$17 \text{ r } 6$$

Point out how this helps us in our new sums ; we can see that there are £4 sh. 4 in sh. 84, and £17 sh. 6 in sh. 346.

Ask how many £ and sh. in sh. 57. When this has been mentally

done, show how it is done by the quick method, explaining that any 1 left over after dividing by 2 is one ten shilling ; so

$$2\cancel{0})\underline{5\cancel{7}}$$
$$2 \text{ r } 17$$

Step 3. Addition of sh. only, giving answers in £ and sh. This is chiefly to give practice in Step 2.

£	sh.	
	10	
	15	
1	5	*Ans.*

$2\cancel{0})\underline{2\cancel{5}}$ 1 r 5

£	sh.	
	19	
	8	
	16	
2	3	*Ans.*

$2\cancel{0})\underline{4\cancel{3}}$ 2 r 3

£	sh.	
	12	
	17	
	6	
	11	
	14	
3	0	*Ans.*

$2\cancel{0})\underline{6\cancel{0}}$ 3 r 0

An exercise of about 10 sums will probably be sufficient.

Step 4. Introduce amounts in £ column, answer not being more than £1,000, and increase gradually to a maximum of 5 items.

£	sh.	
5	14	
+7	10	
13	4	*Ans.*

£	sh.	
6	18	
+	16	
7	14	*Ans.*

$2\cancel{0})\underline{3\cancel{4}}$ 1 r 14

£	sh.	
10	15	
+16	5	
27	0	*Ans.*

$20)\underline{20}$ 1 r 0

£	sh.	
24	13	
43	7	
+ 6	10	
74	10	*Ans.*

$2\cancel{0})\underline{3\cancel{0}}$ 1 r 10

£	sh.	
43	16	
9	8	
73	14	
+ 50	0	
176	18	*Ans.*

$2\cancel{0})\underline{3\cancel{8}}$ 1 r 18

£	sh.	
101	2	
	17	
63	13	
+ 94	10	
260	2	*Ans.*

$2\cancel{0})\underline{4\cancel{2}}$ 2 r 2

£	sh.		£	sh.	
179	11		607	2	
14	12		81	1	
150	0		183	15	
143	18		4	6	
+ 9	9		+ 21	12	
497	10	*Ans.*	897	16	*Ans.*
	$2\cancel{0})5\cancel{0}$			$2\cancel{0})3\cancel{6}$	
	2 r 10			1 r 16	

Give an exercise of at least 20 sums based on these examples. *Note* that when dealing with £ and sh. a unit number of shillings is not preceded by a 0 in the tens column of the shillings.

SUBTRACTION

In this, it must be stressed that when borrowing from the £ column it adds 20 shillings to the shillings column. The top line should be gradually increased.

Step 1. Subtraction of shillings from pounds and shillings.

£	sh.		£	sh.		£	sh.		£	sh.	
$\cancel{1}$	$\cancel{10}$		$\cancel{2}$	14		$\cancel{3}$	3		$\cancel{5}$	0	
−	18		−	19		−	17		−	4	
	12	*Ans.*	1	15	*Ans.*	2	6	*Ans.*	4	16	*Ans.*

An exercise of 10 sums should be sufficient.

Teacher should give more examples as those shown above. Children should be encouraged to dispense with the crossing, and writing in of the new figure in this class.

Step 2. Subtraction of £ and sh. from £ and sh.

£	sh.	£	sh.	£	sh.	£	sh.
2	5	6	12	10	8	23	16
−1	15	−4	10	− 8	10	−14	18

£	sh.	£	sh.	£	sh.	£	sh.
74	2	100	15	253	0	561	7
−60	14	− 97	18	−149	11	−295	13

Teacher to make more examples, building up gradually to £1,000.

MULTIPLICATION

The £ in the top line should not exceed three figures, nor the multiplier 2 figures. Start first with a period on short multiplication to revise and stress the method of change of sh. into £.

Step 1. Short multiplication of £ and sh.

£	sh.	
6	16	
	$\times 7$	
47	12	*Ans.*
5	$2\cancel{0})11\cancel{2}$	
42	5 r 12	
47		

£	sh.	
12	9	
	$\times 9$	
112	1	*Ans.*
4	$2\cancel{0})8\cancel{1}$	
108	4 r 1	
112		

£	sh.	
47	18	
	$\times 8$	
383	4	*Ans.*
7	$2\cancel{0})14\cancel{4}$	
376	7 r 4	
383		

£	sh.	
163	7	
	$\times 6$	
980	2	*Ans.*
2	$2\cancel{0})4\cancel{2}$	
978	2 r 2	
980		

Give an exercise of at least 10 sums.

Step 2. Long multiplication of £ and sh. Increase gradually the size of the numbers.

(*a*) Multipliers below 20.

(i)

£	sh.	
13	12	
	$\times 13$	
176	16	*Ans.*
7	$2\cancel{0})15\cancel{6}$	
39	7 r 16	
130		
176		

£	sh.	
27	8	
	$\times 18$	
493	4	*Ans.*
7	$2\cancel{0})14\cancel{4}$	
216	7 r 4	
270		
493		

(ii)

£	sh.	
154	11	
	×19	
2936	9	*Ans.*
10	99	
1386	110	
1540	2$\cancel{0}$)20$\cancel{9}$	
2936	10 r 9	

Give 10 sums like (i) and 6 like (ii).

(*b*) Multipliers under 100.

£	sh.		£	sh.	
68	19		591	17	
	×47			×78	
3240	13	*Ans.*	46164	6	*Ans.*
44	133		66	136	
476	760		4728	1190	
2720	2$\cancel{0}$)89$\cancel{3}$		41370	2$\cancel{0}$)132$\cancel{6}$	
3240	44 r 13		46164	66 r 6	

Give about 10 sums like this.

DIVISION

The number in the divisor should not exceed three figures (nor the number of £'s in the answer). Begin with short division.

Step 1. Short division of £ and sh.

£	sh.		£	sh.		£	sh.	
	54			91			144	
6)38	$\cancel{14}$		7)53	$\cancel{11}$		12)631	$\cancel{4}$	
6	9	*Ans.*	7	13	*Ans.*	52	12	*Ans.*

Teacher must make more examples to give practice in carrying £ to sh.

Step 2. Long division of £ and sh. without a remainder, gradually increasing the size of the number. The changing of £'s into sh. must be done in the column where the remainder stands

and the resulting shilling be labelled, then transferred to the shillings column. Give class at least 10 examples graded as below.

(*a*) *Divisors of* 13 *to* 19 *only.*

```
      £        sh.                   £        sh.
      14       18  Ans.              29       16  Ans.
  13)193       14              18)536          8
     13       220                 36         280
     --       ---                 ---        ---
      63      234                 176        288
      52       13                 162         18
      --      ---                 ---        ---
      11      104                  14        108
     ×20      104                 ×20        108
     ---      ---                 ---        ---
    220 sh.                      280 sh.
              ---                            ---
```

(*b*) *Divisors over* 19.

```
      £        sh.                   £        sh.
     337       19  Ans.              68       19  Ans.
  19)6421       1              47)3240        13
     57       360                 282        880
     --       ---                 ---        ---
      72      361                 420        893
      57       19                 376         47
     ---      ---                 ---        ---
     151      171                  44        423
     133      171                 ×20        423
     ---      ---                 ---        ---
      18                         880 sh.
              ---                            ---
     ×20
     ---
    360 sh.

      £        sh.                   £        sh.
      37        5  Ans.             109        7  Ans.
  75)2793      15              83)9076         1
     225      360                 83         580
     ---      ---                 --         ---
      543     375                  776       581
      525     375                  747       581
      ---     ---                  ---       ---
       18                           29
              ---                            ---
      ×20                          ×20
      ---                          ---
    360 sh.                      580 sh.
```

(*c*) *Divisors of 3 figures.*

```
           £        sh.                       £         sh.
          151        19  Ans.                105          6  Ans.
     116)17626        4                 236)24850        16
         116       2200                     236        1400
         ---       ----                     ---        ----
          602      2204                     1250       1416
          580       116                     1180       1416
          ---       ---                     ----       ----
           226     1044                       70
           116     1044                      ×20       ----
           ---     ----                     ----
           110                              1400 sh.
           ×20     ----
          ----
          2200 sh.
```

(Note medial zero in £s answer and omission of the writing of the initial zero in sh. answer.)

Step 3. Long division of £ and sh. with a remainder.

Work an example, which has a remainder, on blackboard with class. The remainder remains in sh. Do not introduce cents with £ and sh.

```
               £          sh.
              185          2 r  sh.19  Ans.
e.g.       37)6849        13
              37          80
              --          --
              314         93
              296         74
              ---         --
               189        19 sh.
               185
               ---
                 4
               ×20
               ---
                80 sh.
```

If the remainder is 20 or more shillings, the remainder is written in £ and sh.

e.g.

$$
\begin{array}{rrr}
 & \pounds & \text{sh.} \\
 & 60 & 12 \text{ r } \pounds 2 \text{ sh.}11 \;\; \textit{Ans.} \\
\cline{2-3}
156) & 9456 & 3 \\
 & 936 & 1920 \\
\cline{2-3}
 & 96 & 1923 \\
 & \times 20 & 156 \\
\cline{2-3}
 & 1920 \text{ sh.} & 363 \\
 & & 312 \\
\cline{3-3}
 & & 51 \text{ sh.} = \pounds 2 \text{ sh.}11
\end{array}
$$

Give at least 6 examples on this step.

PROFIT AND LOSS

By this stage the class will have done the mechanical working of money totalling up to sh. 1,000 and the work in £ and sh., so the mechanical work of this topic should present no difficulties.

Step 1. Revision of Profit and Loss of Class 4.

Revise what the class learnt on this topic in Class 4 :

1. What profit is, and how it is known to be profit ;
2. What loss is, and how it is known to be loss ;
3. Cost price ;
4. Selling price ;
5. Formula for finding profit ;
6. Formula for finding loss ;

by simple mental arithmetic and questioning.

E.g. (1) Musa bought a jembe for sh. 5 and sold it for sh. 6.50. What profit did he make?—sh. 1.50.

Question : ' How did I know that he made a profit? '
Answer : ' Because he sold it for more than he paid.'
Question : ' How did you find the profit? '
Answer : ' S.P. – C.P.'

E.g. (2) Mary bought a basket for sh. 4.50 and sold it later for sh. 4. Did she make a profit or a loss?—Loss.

Question : ' How do you know it was a loss? '
Answer : ' Because she sold it for less than she paid.'

Question : 'What was her loss?'
Answer : '50 cents.'
Question : 'How did you find the loss?'
Answer : 'C.P. – S.P.'

Step 2. Practical work on revision of Class 4.

Examples should gradually be made more difficult.

E.g. (1) Joseph bought 6 cows for sh. 383, and sold them for sh. 401. What was his profit? Make 5 examples asking profit, and 5 asking loss.

Step 3. Finding profit or loss with internal multiplication.

(*a*) Write the following on the blackboard :

I sold 12 cows for sh. 153 each. How much did I get?

Question : 'What have I to find?'
Answer : 'Selling price of 12 cows'
Question : 'How will I find the answer?'
Answer : 'By multiplying sh. 153 by 12.'

With the class, work on the blackboard as follows :

S.P. = sh. 153 × 12 = sh. 1836. Leave on the blackboard.

Then change the question on the blackboard to read :

I sold 12 cows for sh. 153 each. I paid sh. 2,000 for the twelve. How much loss did I make?

Question : 'How will we find the loss?'
Answer : 'C.P. – S.P.'
Question : 'Do we know the selling price?'
Answer : 'Yes. Sh. 1836.' This is already written on the blackboard.
Question : 'Do we know the C.P.?'
Answer : 'Yes. Sh. 2,000.'

Write the new fact on the blackboard so that the sum now reads :

S.P. = sh. 153 × 12 = sh. 1836
C.P. = sh. 2,000

Ask a child to find the loss. Write on the blackboard to complete the sum :

∴ Loss = sh. 2,000 – 1,836 = sh. 164

(*b*) Write the following sum on the blackboard :

Charles bought 8 sheets of mabaati for sh. 25 each. He sold them all for sh. 244. Did he make a profit or a loss? How much?

Question : ' Before we can find a profit or a loss, what must we know? '
Answer : ' The C.P. and the S.P.
Question : ' Do we know the C.P.? '
Answer : ' No.'
Question : ' How will we find it? '
Answer : ' By multiplying sh. 25 by 8.'

Ask a child to work it as you write on the blackboard :

$$\text{C.P.} = \text{sh. } 25 \times 8 = \text{sh. } 200$$

Question : ' Do we know the S.P.? '
Answer : ' Yes. Sh. 244.'

Write on the blackboard under C.P. statement :

$$\text{S.P.} = \text{sh. } 244$$

Question : ' Did Charles make a profit or a loss? '
Answer : ' A profit.'
Question : ' How do you know? '
Answer : ' Because the S.P. is greater than the C.P.'

Ask a child to work the sum, as you write on the blackboard, to complete the sum to read :

$$\text{C.P.} = \text{sh. } 25 \times 8 = \text{sh. } 200$$
$$\text{S.P.} = \text{sh. } 244$$
$$\therefore \text{ Profit} = \text{sh. } 244 - 200 = \text{sh. } 44$$

Step 4. Practical work on Step 3.

Give at least 10 examples asking for profit, and 10 asking for loss, including internal multiplication, similar to those given above.

Step 5. Finding Cost Price, given Selling Price and Profit or Loss.

Lead children to find the formulae for themselves.

(*a*) Given profit.

Write the following on the blackboard:

John made a profit of sh. 23 by selling a bicycle for sh. 320. How much did he pay for it?

Question : 'What have we to find?'
Answer : 'The cost price.'
Question : 'Did he pay more or less than he received for it?'
Answer : 'Less.'
Question : 'How do you know?'
Answer : 'Because he made a profit.'
Question : 'What was his profit?'
Answer : 'Sh. 23.'

Write on the blackboard:

Profit = sh. 23

Question : 'How much did he receive when he sold it?'
Answer : 'Sh. 320.'

Write on the blackboard under the profit statement:

S.P. = sh. 320

Question : 'If he paid less for it, how will we find how much he paid?'
Answer : 'By subtracting the profit from the S.P.'

Write the rule on the blackboard: C.P. = S.P. − Profit. Ask a child to work the sum, as you write on the blackboard, so that the sum reads:

Profit = sh. 23
S.P. = sh. 320
∴ C.P. = sh. 320 − 23 = sh. 297

(*b*) Given loss.

Write on the blackboard:

Peter sold some cotton for sh. 87. He lost sh. 13. How much did he pay for the cotton?

Question as for previous example (remember it is a loss), so

that class give formula C.P. = S.P. + Loss. Write this rule on the blackboard. The sum on the blackboard should read :

Loss = sh. 13

S.P. = sh. 87

∴ C.P. = sh. 87 + 13 = sh. 100

Step 6. Practical work on Step 5.

Give at least 10 examples giving profit, and at least 10 giving loss, asking class to find C.P. Examples should be worded similarly to those given above.

Step 7. Finding selling price, given profit or loss.

Write the following on the blackboard :

Musa bought a bicycle for sh. 342. He made a profit of sh. 39. What did he sell it for?

Question : 'Did Musa make a profit or a loss?'
Answer : 'A profit of sh. 39.'

Write on the blackboard :

Profit = sh. 39

Question : 'Do we know what he received for it?'
Answer : 'No.'
Question : 'Do we know what he paid for it?'
Answer : 'Yes. Sh. 342.'

Write under Profit statement on the blackboard :

C.P. = sh. 342

Question : 'Did he receive more or less for it than he paid?'
Answer : 'More.'
Question : 'How do you know.'
Answer : 'Because he made a profit'
Question : 'If he received more, how will we find how much the S.P. was?'
Answer : 'By adding the C.P. and the profit.'

Write on the blackboard :

S.P. = C.P. + Profit

Ask a child to work the sum, while you complete the sum on the blackboard, to read:

Profit = sh. 39
C.P. = sh. 342
∴ S.P. = sh. 342 + 39 = sh. 381

(*b*) Given loss:

Write the following on the blackboard:

Paul lost sh. 21 when he sold a cow for which he had paid sh. 102. What was the selling price?

Question as for the previous example (remember it is a loss), so that class give formula S.P. = C.P. − Loss. Write this rule on the blackboard. Sum should read:

Loss = sh. 21
C.P. = sh. 102
∴ S.P. = sh. 102 − 21 = sh. 81

Step 8. Practical work on Step 7.

Give at least 10 examples giving profit, and at least 10 giving loss, asking class to find S.P. Examples should be worded similarly to those given above.

LINEAR MEASURE

Aim

The mechanical calculation of measure is to be completed by including the mile unit in multiplication and division, and by the mechanical process of dividing one measure into another.

Revise all tables and add '1 ml. = 1760 yd.'

MULTIPLICATION

Step 1. Short multiplication of ch. yd. ft. with ml. in answer.

This is merely a transitional step to facilitate the introduction of miles in multiplication.

	ml.	ch.	yd.	ft.	
(*a*)		43	18	2	
				× 12	
	6	46	4	0	*Ans.*
		10	8	3)24	
		516	216	8 r 0	
		6 r 46	10 r 4		
		80)526	22)224		
		480	220		
		46	4		

	ml.	ch.	yd.	ft.	
(*b*)		52	0	2	
				× 11	
	7	12	7	1	*Ans.*
		7 r 12	7	3)22	
		80)572		7 r 1	
		560			
		12			

Demonstrate with class co-operation. Give the class 5 examples of (*a*) and 5 of (*b*) to do on their own. Encourage all pupils, as far as possible, to do the changing from feet to yards mentally, and to dispense with the underline working for the feet column.

Step 2. Short multiplication of ml. ch. yd.

The ' feet ' unit is now omitted. See introduction to ' chains ', Class 4.

	ml.	ch.	yd.	
(*a*)	4	62	18	
			× 9	
	43	5	8	*Ans.*
	7	7	7 r 8	
	36	558	22)162	
	43	7 r 5	154	
		80)565	8	
		560		
		5		

	ml.	ch.	yd.	
(*b*)	4	0	21	
			× 12	
	48	11	10	*Ans.*
		11	11 r 10	
			22)252	
			22	
			32	
			22	
			10	

10 examples of each to be done by class.

Step 3. Long multiplication of ch. and yd. with ml. in answer.

ml.	ch.	yd.	
	48	20	
		×13	
7	75	18	*Ans.*
	11	11 r 18	
	144	22)260	
	480	22	
	7 r 75	40	
	80)635	22	
	560	18	
	75		

Give 10 examples with multipliers less than 20. This will establish the method in a tidy way. Among your examples have three or four with 0 yd. or 1 yd.

Step 4. Long multiplication of ml. ch. yd.

At this stage it will be necessary to give the class a fresh rule for spacing the units in their sums, in order to ensure neat and legible underline working. If the class are still using squared books (they should *not* be in Class 5) make your new rule '*seven* empty squares between ml. and ch., and ch. and yd.'. If the class has changed to narrow-lined books, stress the importance of leaving sufficient room to deal with six or even seven numbers in a row in the underline working. Go round and see that this is being done.

Note: There is never any need to give sums with multipliers of more than two digits in mensuration sums (i.e. length, weight, etc.).

Here follows a model grading of examples. Notice that one sum of each type is given, but *you must make at least another five of each* for class work. Follow the grading given. In sections (*c*) and (*e*) you may increase the multipliers gradually to 99. Section (*f*) is an example of a medial 0. At least one sum of this type should be included in sections (*a*) to (*e*), obeying the rules given for each section. Give also one or two examples where there is a

single digit in the chains column and no carrying from chains to miles.

(*a*) One digit in miles; multiplier in tens. (For revision of rule that two digits are always multiplied by one, not one by two.)

ml.	ch.	yd.
4	26	13
		×19

(*b*) Two digits in miles—multiplier in teens.

ml.	ch.	yd.
25	63	19
		×14

(*c*) Two digits in miles—multiplier 20 and over.

ml.	ch.	yd.
18	42	19
		×28

(*d*) Three digits in miles—multiplier in teens.

ml.	ch.	yd.
136	47	16
		×17

(*e*) Three digits in miles—multiplier 20 and over.

ml.	ch.	yd.
137	53	17
		×29

(*f*) Medial 0 (see beginning of Step 4).

ml.	ch.	yd.
182	0	12
		×43

This example is suitable for inclusion in section (*e*). The class should be able to deal with medial 0, and no demonstration should be given. The teacher should, however, realise that it is a possible source of difficulty and be ready to deal with any troubles that arise.

DIVISION

Step 1. Division of ml. ch. yd. One digit only in each unit of answer. No remainders. Divisors gradually increased from 13–99.

In Steps 1–5 inclusive, the easiest way to make examples for the class is to take a length measurement which obeys the 'rules for the answer' given in each step (e.g., in this step the answer must have one figure only in ml. ch. and yd.), multiply it by a number which obeys the rules given for the divisor in each step (always the same—i.e., between 13 and 99) and give the result of this sum to the class to divide. For example, to provide practice in Step 1, let us take a measurement with one figure in each unit.

	ml.	ch.	yd.
	9	6	7
and multiply it by			×13
we get the answer	118	2	3

and so we make our first sum for the class :

	ml.	ch.	yd.
13)	118	2	3

Demonstrate also :

$$\begin{array}{rlll} & \text{ml.} & \text{ch.} & \text{yd.} \\ 25) & \overline{102} & \overline{67} & \overline{2} \end{array}$$

The class should do 10 examples themselves when the teacher has made them up on these lines.

Step 2. Division with 2 figures in yards column of answer.

$$\begin{array}{rlll} & \text{ml.} & \text{ch.} & \text{yd.} \\ 37) & \overline{263} & \overline{7} & \overline{21} \end{array}$$

Make 10 examples for the class to do.

Step 3. Division with two figures in ch. and yd. columns of answer.

$$\begin{array}{rlll} & \text{ml.} & \text{ch.} & \text{yd.} \\ 41) & \overline{350} & \overline{34} & \overline{15} \end{array}$$

10 examples to be worked by the class.

Step 4. (*a*) Two figures in *answer* throughout.
(*b*) Three figures in miles, two each in yards and chains of the answer.

$$\begin{array}{rlll} & \text{ml.} & \text{ch.} & \text{yd.} \\ (a) \quad 53) & \overline{765} & \overline{53} & \overline{12} \end{array}$$

$$\begin{array}{rlll} & \text{ml.} & \text{ch.} & \text{yd.} \\ (b) \quad 38) & \overline{4692} & \overline{69} & \overline{14} \end{array}$$

Make 5 examples of (*a*) and 5 of (*b*) for the class to do.

Note: When making your examples, keep the divisors as a rule fairly *low*. Some may have a higher divisor for the benefit of the brighter children.

Step 5. Division with 0 difficulties.

Most of the children should at this stage be able to cope with any 0 difficulties which arise. If any are still in doubt, take some of the examples from Long Division of Length, Class 4, Step 6, work them with the backward ones, and then give some examples including miles. Here are the three new types which they will meet :

(*a*) No carrying ml. to ch. Ch. less than divisor, so 0 in ch. column of answer.

```
        ml.       ch.        yd.
         19         0         16  Ans.
      ----------------------------
    35)665         25         10
        35        ×22        550
       ---        ---        ---
       315         50        560
       315        500        35
       ---        ---        ---
                  550 yd.    210
                             210
                             ---
```

(*b*) Miles less than divisor so blank in miles column of answer.

```
        ml.       ch.        yd.
                   53         19  Ans.
      ----------------------------
    45)30          23         19
       ×80       2400        836
      ----       ----        ---
      2400 ch.   2423        855
                 225         45
                 ---         ---
                  173        405
                  135        405
                  ---        ---
                   38
                  ×22
                  ---
                   76
                  760
                  ---
                  836 yd.
```

(*c*) Three figures in miles column of answer with 0 in tens place, e.g. 103.

ml.	ch.	yd.
103	29	13 *Ans.*
23)2377	40	13
23	640	286
··77	680	299
69	46	23
8	220	69
× 80	207	69
640 ch.	13	—
	× 22	
	26	
	260	
	286 yd.	

All the class should do 3 examples of each of these three types.

Step 6. Long division with remainders.

(*a*) Yards in answer, and remainder in yards.

ml.	ch.	yd.
17	73	9 r 2 yd. *Ans.*
39)698	63	1
39	2800	352
308	2863	353
273	273	351
35	133	2
× 80	117	
2800 ch.	16	
	× 22	
	32	
	320	
	352 yd.	

(*b*) Yards in answer, with remainder in ch. and yd.

ml.	ch.	yd.
3	25	11 r 1 ch. 6 yd. *Ans.*
29)96	20	17
87	720	330
9	740	347
×80	58	29
720 ch.	160	57
	145	29
	15	28 = 1 ch. 6 yd.
	×22	
	30	
	300	
	330 yd.	

(*c*) 0 yd. in answer, but with remainder in yd. or ch. and yd.

ml.	ch.	yd.
2	56	0 r 3 ch. 15 yd. *Ans.*
84)226	67	15
168	4640	66
58	4707	81 = 3 ch. 15 yd.
×80	420	66 *
4640 ch.	507	15
	504	
	3	
	×22	
	66 yd.	

* The explanation of this step is done by the same system as that of Long Division of Length, Class 4, Step 7(*d*).

The teacher should make at least 3 examples of each type (*a*)–(*c*) above for the children to do. Make them as in Steps 1–5 above, but when your multiplication sum is completed, add in a suitable quantity to provide the remainder.

The teacher may have noticed that remainders sometimes appear expressed as fractions. This is a complication which *must not* be presented to the children at this level, however.

At this point, at least two lessons should be spent on general revision practice by the children of the four rules in length. Particular attention should be paid to the addition and subtraction of miles, chains and yards which was introduced in Class 4; but work in all quantities down to inches should be set.

DIVISION OF LENGTH BY LENGTH (*Optional*)

This is a mechanical process used normally only in the solution of problems, but it should be taught thoroughly *as a mechanical process* before any problems are attempted. The general rule for working the sums is simple:

> 'Reduce both given lengths (i.e. change them) into the lowest unit and divide. The answer is *not* a length but simply a *number*.'
>
> (Lowest unit—i.e., if one length is given in yd. and ft. and the other in in., both must be changed to in.)

Introduction

Revise the meaning of the division sign using an explanation like this: '12 ÷ 2 means 12 divided by 2 or *how many 2s in* 12. Similarly 10 in. ÷ 2 in. means how many lengths of 2 in. are there in a length of 10 in., and 1 ft. 9 in. ÷ 3 in. means how many lengths of 3 in. are there in 1 ft. 9 in. We often need to find the answer to a sum of this kind—for instance, how many rows of potatoes we can plant in a plot if the plot is 18 yd. wide and the rows must be 1 ft. 9 in. apart. We have to ask ourselves "How many times 1 ft. 9 in. are there in 18 yd.", or, we have to do this sum' . . . (write on blackboard) '18 yd. ÷ 1 ft. 9 in.'

'All of you measure a line on your desks 1 ft. 6 in. long. Now count how many times you can measure 3 in. along that line.' (Get a boy to tell you the answer—6 times.) 'Now look at the blackboard.'

1 ft. 6 in. = 18 in.

(Build up on the blackboard by questions.)

$$3\overline{)18}$$
$$6$$

'So, if we change the measurements to the same unit, and divide, we get the same answer as when we measured the lines

on the desks. Let us try a harder one.' (Call a boy to the blackboard and let him draw a line 2 ft. 9 in. long. Then call another pupil to divide the line with your ruler into lengths of 11 in. Then say . . .) 'Our line was 2 ft. 9 in. long.' (Write 2 ft. 9 in. on blackboard.) 'We divided it into lengths of 11 in.' (Put 11 in. on blackboard with ÷ sign.) 'Now let us change the 2 ft. 9 in. to in. How do we do it?' (When the answer comes, do the sum on the blackboard.) 'We divide 33 by 11 and we get the same answer as ——— got by measuring.'

(Your blackboard should now look like this.)

$$
\begin{array}{rrcr}
\text{ft.} & \text{in.} & & \text{in.} \\
2 & 9 & \div & 11 \\
\times 12 & 24 & & \\
\hline
24 \text{ in.} & 11\overline{)33} \text{ in.} & & \\
 & 3 \text{ times} & &
\end{array}
$$

After this, demonstrate how a length of yd. ft. and in. may be divided into a number of smaller lengths. (These should be less than 1 ft. at the present stage.)

$$
\begin{array}{lrrcr}
\text{yd.} & \text{ft.} & \text{in.} & & \text{in.} \\
2 & 1 & 4 & \div & 8 \\
3 & 6 & 84 & & \\
\overline{6} \text{ ft.} & \overline{7} & 8\overline{)88} & & \\
 & \times 12 & \overline{11} \text{ times} & & \\
 & \overline{84} \text{ in.} & & &
\end{array}
$$

Step 1. Practice.

(*a*) Ft. in. ÷ in.

At least 5 examples like the following:

ft.	in.		in.	ft.	in.		in.	
2	11	÷	7	1	9	÷	3	etc.

(*b*) Yd. ft. in. ÷ in., including 0 quantities.

yd.	ft.	in.		in.	yd.	ft.	in.		in.
3	2	8	÷	10	4	1	0	÷	4
yd.	ft.	in.		in.	yd.	ft.	in.		in.
3	0	4	÷	7	2	2	3	÷	11

At least 10 examples to be made in (*b*) for the class to do.

Step 2. Yd. ft. in. ÷ ft. in.

Revise the rule and make sure the class know that both quantities have to be 'reduced'. Do not have more than 10 yd. in the dividend. Now that the two quantities have to be reduced, it will be found much neater if the division sum is done separately beneath, as shown here:

yd.	ft.	in.		ft.	in.
9	1	0	÷	1	9
3	27	336		×12	12
27 ft.	28	336		12 in.	21
	×12				
	336 in.				

16 times *Ans.*

$$\begin{array}{r} 16 \\ 21\overline{)336} \\ 21 \\ \hline 126 \\ 126 \\ \hline \ldots \end{array}$$

Where the divisor is a simple quantity in ft. and in., many pupils will be able to do the reduction mentally. Allow those who can to do so.

Make at least 5 examples for the class to do.

Step 3. Yd. ft. in. ÷ yd. ft. in., up to 21 yd. in dividend.

Little demonstration should be needed. Make at least 5 examples like the following model for the class to do by themselves. Include in your examples some in which either the divisor or dividend or both have 0 ft. or in. or both.

yd.	ft.	in.		yd.	ft.	in.
20	2	6	÷	1	1	2
×3	60	744		×3	3	48
60 ft.	62	750		3 ft.	4	50
	×12				×12	
	744 in.				48 in.	

15 times *Ans.*

$$\begin{array}{r} 15 \\ 50\overline{)750} \\ 50 \\ \hline 250 \\ 250 \\ \hline \ldots \end{array}$$

Encourage mental working of small reductions where possible.

Step 4. Sums as in Steps 1, 2 and 3, but with remainders.

Here you must make sure that the class knows what the remainder is; teach them the rule that 'The remainder is always a *length* left over'. This step is best introduced practically by measuring a wall, door, the floor, etc. Do it yourself first to ensure that, by using a foot-rule or a yard stick, there is in fact something left over when you have used the ruler as often as possible. Let the class do some similar practical work, demonstrating afterwards not less than two sums on the blackboard. Stress always that the remainder is a length and must be written as a length in the proper way.

	yd.	ft.	in.		yd.	ft.	in.
(i)	14	2	11	÷	2	1	3
	×3	42	528		×3	6	84
	42 ft.	44	539		6 ft.	7	87
		×12				×12	
		528 in.				84 in.	

6 times r 1 ft. 5 in. *Ans.*

87)539
522
17 in. remainder

	yd.	ft.	in.		yd.	ft.	in.
(ii)	21	2	11	÷	3	1	2
	×3	63	780		×3	9	120
	63 ft.	65	791		9 ft.	10	122
		×12				×12	
		780 in.				120 in.	

6 times r 1 yd. 2 ft. 11 in. *Ans.*

122)791
732
59 in. remainder

Change to yd. ft. in. mentally.

You should make at least 5 examples for the class to do in this section. The last step in the division (changing a remainder to ft. in., or yd. ft. in.) should be done mentally as stated; but any

backward pupils may be allowed to work out the reduction by the written method.

In all the remaining steps, one model alone is given, but the teacher should make his own examples for the class to do—not less than 2 per step. 0 quantities should be introduced into some examples at all steps.

Step 5. Ch. yd. ft. ÷ yd. ft.

ch.	yd.	ft.		yd.	ft.
35	14	0	÷	18	2
× 22	770	2352		× 3	54
70	784	2352		54 ft.	56
700	× 3				
770 yd.	2352 ft.				

42 times *Ans.*

$$56\overline{)2352}$$

224

112

112

Step 6. Ch. yd. ft. ÷ ch. yd. ft.

Note: Do not have more than 10 ch. in your divisor. To do so is to make sums unnecessarily unwieldy.

ch.	yd.	ft.		ch.	yd.	ft.
68	7	0	÷	7	13	0
× 22	1496	4509		× 22	154	501
136	1503	4509		154 yd.	167	501
1360	× 3				× 3	
1496 yd.	4509 ft.				501 ft.	

9 times *Ans.*

$$501\overline{)4509}$$

4509

Step 7. Sums as Steps 5 and 6 but with remainders.

Revise the rule that a remainder is a 'length left over' and be sure the class realise that the remainder in this step is 'feet'.

All the class may be allowed to 'reduce' the remainder to its proper statement in writing, but if any *can* do it mentally they should be encouraged.

ch.	yd.	ft.		ch.	yd.	ft.
79	21	2	÷	9	14	1
×22	1738	5277		×22	198	636
158	1759	5279		198 yd.	212	637
1580	×3				×3	
1738 yd.	5277 ft.				636 ft.	

8 times r 2 ch. 17 yd.

637)5279
5096
3)183 ft.
2 ch. 17 yd.
22)61 yd.
44
17 yd. r

Step 8. Ml. ch. yd. ÷ ch. yd.

Sufficient practice can be given with a limit of 5 miles in the dividend and 25 chains in the divisor. Do not go above these figures when making your own examples.

ml.	ch.	yd.		ch.	yd.
4	3	5	÷	24	19
×80	320	7106		×22	528
320 ch.	323	7111		48	547
	×22			480	
	646			528 yd.	
	6460				
	7106 yd.				

13 times *Ans.*

547)7111
547
1641
1641

Step 9. Ml. ch. ÷ ml. ch. (limit 100 ml. in dividend).

ml.	ch.		ml.	ch.
93	71	÷	2	43
×80	7440		×80	160
7440 ch.	7511		160 ch.	203

```
     37 times  Ans.
203)7511
    609
    1421
    1421
```

Step 10. Sums as in Step 8 and Step 9 but with remainders.

	ml.	ch.	yd.		ch.	yd.
(*a*)	4	67	13	÷	23	15
	×80	320	8514		×22	506
	320 ch.	387	8527		46	521
		×22			460	
		774			506	
		7740				
		8514				

```
     16 times r 8 ch. 15 yd.  Ans.
521)8527
    521
    3317
    3126
       8 ch. 15 yd.
22)191 yd.
   176
    15 yd.
```

(b)

ml.	ch.		ml.	ch.
97	31	÷	4	8
× 80	7760		× 80	320
7760 ch.	7791		320 ch.	328

$$\begin{array}{r} 23 \text{ times r 3 ml. 7 ch. } \textit{Ans.} \\ 328\overline{)7791} \\ \underline{656} \\ 1231 \\ \underline{984} \\ 80\overline{)247} \\ \overline{3 \text{ ml. r 7 ch.}} \end{array}$$

The teacher should make *at least* 3 examples of each of the types demonstrated in Steps 5 to 9, and at least 5 of Step 10 for the children to do themselves.

CAPACITY

Aim

To introduce long multiplication and long division.

Step 1. Revision of the four rules for de. gall. and pt. as learnt in Class 3. (probably 2 periods.)

(a)

de.	gall.	pt.
6	3	7
+ 4	0	2
11	0	1
1	1	$8\overline{)9}$
10	3	1 r 1
11	$4\overline{)4}$	
	1 r 0	

(b)

de.	gall.	pt.
7	0	2
−4	3	5
2	0	5

(c)

de.	gall.	pt.
4	3	4
		×9
43	3	4
7	4	8)36
36	27	4 r 4
43	4)31	
	7 r 3	

(d)

	de.	gall.	pt.
		30	48
12)	43	~~2~~	~~0~~
	3	2	4

Step 2. Introduce long multiplication of gallons and pints (with debes in the answers). At this stage it should be made clear that the smaller figure (i.e. the single digit figure) is used as the multiplier, irrespective of the fact that it is in the upper line rather than the lower, e.g. in example 1 below, say 6 times 13, and *not* 13 times 6.

de.	gall.	pt.
	2	6
		×13
8	3	6
8	9	8)78
	26	9 r 6
	4)35	
	8 r 3	

de.	gall.	pt.
	3	1
		×15
11	2	7
11	1	8)15
	45	1 r 7
	4)46	
	11 r 2	

de.	gall.	pt.
	1	7
		×21
9	3	3
9	18	8)147
	21	18 r 3
	4)39	
	9 r 3	

de.	gall.	pt.
	2	6
		×37
25	1	6
25	27	8)222
	74	27 r 6
	4)101	
	25 r 1	

Give an exercise of about 10 sums.

Step 3. Introduce debes into the top line, first as a single digit, then later as a 2 digit figure. In this latter case the lower

line figure will normally be used as the multiplier for the debe column, i.e. in example*, say 3 times 14, then 10 times 14.

de.	gall.	pt.	
2	3	4	
		×13	
37	1	4	
11	6	8)52	
26	39	6 r 4	
37	4)45		
	11 r 1		

de.	gall.	pt.	
3	1	4	
		×14	
47	1	0	
5	7	8)56	
42	14	7 r 0	
47	4)21		
	5 r 1		

de.	gall.	pt.	
2	3	4	
		×24	
69	0	0	*Ans.*
21	12	8)96	
48	72	12 r 0	
69	4)84		
	21 r 0		

de.	gall.	pt.	
5	1	7	
		×43	
235	40	5	*Ans.*
20	37	8)301	
215	43	37 r 5	
235	4)80		
	20 r 4		

de.	gall.	pt.	
*14	2	1	
		×13	
188	3	5	*Ans.*
6	1	8)13	
42	26	1 r 5	
140	4)27		
188	6 r 3		

de.	gall.	pt.	
23	0	7	
		×15	
348	1	1	*Ans.*
3	4)13	8)105	
115	3 r 1	13 r 1	
230			
348			

de.	gall.	pt.	
12	3	6	
		×23	
297	2	2	*Ans.*
21	17	8)138	
276	69	17 r 2	
297	4)86		
	21 r 2		

de.	gall.	pt.	
32	1	0	
		×26	
838	2	0	*Ans.*
6	4)26		
192	6 r 2		
640			
838			

Give an exercise of at least 5 sums.

Step 4. Long division of de. gall. and pt.

(*a*) Without remainder.

```
       de.      gall.     pt.
        5        1         4  Ans.
     ______________________
  13)69          3         4
     65         16        48
     --         --        --
      4         19        52
     ×4         13        52
     --         --        --
     16 gall.    6
                ×8
                --
                48 pt.
```

```
       de.      gall.     pt.
        5        1         2  Ans.
     ______________________
  27)143         1         6
     135        32        48
     ---        --        --
       8        33        54
      ×4        27        54
      --        --        --
      32 gall.   6
                ×8
                --
                48 pt.
```

```
       de.      gall.     pt.
        2        3         7  Ans.
     ______________________
  16)47          2         0
     32         60       112
     --         --       ---
     15         62       112
     ×4         48       112
     --         --       ---
     60 gall.   14
                ×8
               ---
               112 pt.
```

(*b*) With remainder.

```
      de.   gall.   pt.
       1     0      2 r 7 pt.  Ans.
     _________________
  13)14      0      1
     13      4     32
     --      -     --
      1      4     33
     ×4     ×8     26
     --     --     --
      4     32      7
```

```
      de.   gall.   pt.
       2     1      2 r 3 pt.  Ans.
     _________________
  21)48      2      5
     42     24     40
     --     --     --
      6     26     45
     ×4     21     42
     --     --     --
     24      5      3
            ×8
            --
            40
```

$$
\begin{array}{rrrrl}
 & \text{de.} & \text{gall.} & \text{pt.} & \\
 & 2 & 1 & 3 & \text{r 1 pt. } \textit{Ans.} \\
\hline
25) & 58 & 2 & 4 & \\
 & 50 & 32 & 72 & \\
\cline{2-4}
 & 8 & 34 & 76 & \\
 & \times 4 & 25 & 75 & \\
\cline{2-4}
 & 32 & 9 & 1 & \\
 & & \times 8 & & \\
\cline{3-3}
 & & 72 \text{ pt.} & & \\
\end{array}
$$

Give an exercise of about 10 sums.

WEIGHT

Aim

The working of the four rules will be extended to cover the hundredweight (cwt.) and ton, and to include Long Multiplication and Division.

Introduction and Table Work

Tell the class: 'Certain objects always have their weight recorded in lb. and oz. For instance, the weight of a man or woman is said to be so many pounds and so many ounces. But many things have their weight expressed in a different way. The weight of a large bag of sugar, for instance, is usually stated to be so many *hundredweight*. A hundredweight is equal to 112 lb., and is usually written cwt.'

(Write on blackboard.)

1 hundredweight—1 cwt. = 112 lb.

'Many of you have seen lorries travelling along the roads. You have seen huge lorries made to carry petrol and oil. You may have seen lorries loaded with coffee or cotton. The lorries and also their loads are often very heavy indeed, so that they weigh very many of these hundredweights. Very heavy objects and loads are not only measured in hundredweights; they are also measured in tons. One ton is the same as 20 hundredweight. We always write the word ton in full; there is no short way of writing it. It is short enough already.'

(Blackboard now reads):

1 hundredweight—1 cwt. = 112 lb.
1 ton = 20 cwt.

Get the class to work out quickly the number of lb. in a ton. Then complete the table which in its short form will read:

1 cwt. = 112 lb.
1 ton = 20 cwt. = 2240 lb.

The full table of weight from oz. to tons should now be drilled daily until it is known.

Point out to the class that it is unusual for more than three of these units of weight to be found together and that in fact most weights are stated as *X* lb. *Y* oz.; or *X* tons *Y* cwt.; or in some cases *X* tons, *Y* cwt., *Z* lb. In the last case, if the lb. is equal to $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of a cwt., the weight will be written as (show on blackboard):

tons	cwt.	qtr.
2	4	3*

(* or 2 or 1 as appropriate)

'This "qtr." simply means a quarter of a hundredweight. Now you know what it is if you ever see it written on the side of a lorry or a bus, or a railway waggon. We do not bother with it in our arithmetic books.

'Now let us begin to do some work using these bigger weights.'

ADDITION

Step 1. (*a*) Addition of 2 items of lb., with answers in cwt. and lb.

Demonstrate on the blackboard two or three examples as this one, being careful to elicit from the class the method of changing lb. to cwt. This should be obvious to all children at this stage.

	cwt.	lb.	
		63	
+		72	
	1	23	*Ans.*

$$\begin{array}{r} 1 \\ 112\overline{)135} \\ 112 \\ \hline 23 \end{array}$$

(*b*) Addition of lb. to cwt. and lb.

Demonstrate 2 examples as follows:

cwt.	lb.	
2	82	
+	49	
3	19	*Ans.*
1	1	
	112)$\overline{131}$	
	112	
	19	

cwt.	lb.	
	93	
+3	76	
4	57	*Ans.*
1	1	
	112)$\overline{169}$	
	112	
	57	

(*c*) Addition of cwt. lb. to cwt. lb.

Demonstrate one example as follows:

cwt.	lb.	
2	87	
+1	83	
4	58	*Ans.*
1	1	
	112)$\overline{170}$	
	112	
	58	

Give about 10 sums of each kind. This step should not take more than 2 periods.

Step 2. Addition of 3, then 4, items.

After demonstration of one of each as below, give about 10 of each for the children to do.

cwt.	lb.	
3	87	
4	96	
+ 2	101	
11	60	*Ans.*
2	2	
	112)$\overline{284}$	
	224	
	60	

cwt.	lb.	
4	82	
3	9	
4	76	
+ 5	29	
17	84	*Ans.*
1	1	
	112)$\overline{196}$	
	112	
	84	

Step 3a. 2 items of cwt. lb. with tons in answer.

Revise method of changing each unit into terms of the next higher unit. Obtain method of changing cwt. to ton by reference to the table.

ton	cwt.	lb.	
	19	102	
+	17	43	
1	17	33	*Ans.*
1	1	1	
	36	112)145	
	20)37 *	112	
	1 r 17	33	

* Children should be encouraged to do this mentally in this class.

Step 3b. 3 items of cwt. lb. with ton in answer.

ton	cwt.	lb.	
	16	73	
	19	111	
+	6	54	
2	3	14	*Ans.*
2	2	2	
	41	112)238	
	20)43	224	
	2 r 3	14	

Step 4. 3 items, leading to 4 items, adding ton. cwt. lb.

After demonstration, give practice in both three and four item addition. You can mix these sums together.

ton	cwt.	lb.	
10	13	39	
21	19	47	
+64	8	107	
97	1	81	*Ans.*
2	1	1	
	40	112)193	
	20)41	112	
	2 1	81	

SUBTRACTION

Step 1. The subtraction of cwt. lb. from cwt. lb.

cwt.	lb.	
7	128	
~~8~~	~~16~~	
− 3	100	
4	28	*Ans.*

cwt.	lb.	
11	148	
~~12~~	~~36~~	
− 4	47	
7	101	*Ans.*

Give an exercise of at least 10 sums.

Step 2. Introduction of tons. No carrying from tons.

Give the class the two 'demonstration' sums below to do by themselves, *before* doing them on the blackboard. It will be found that probably most, if not all, can do them—it may be necessary to help the weaker ones. Give an exercise of 10 sums.

ton	cwt.	lb.	
	17	144	
27	~~18~~	~~32~~	
− 13	9	83	
14	8	61	*Ans.*

ton	cwt.	lb.	
	13	121	
38	~~14~~	~~9~~	
− 23	4	59	
15	9	62	*Ans.*

Step 3. The same, with carrying throughout.

ton	cwt.	lb.	
52	28 ~~8~~	129	
~~53~~	~~9~~	~~17~~	
− 46	11	71	
6	17	58	*Ans.*

ton	cwt.	lb.	
71	30 ~~10~~	149	
~~72~~	~~11~~	~~37~~	
− 69	17	93	
2	13	56	*Ans.*

Give an exercise of at least 10 sums.

Step 4. As Step 3, with the difficulty of carrying across 0.

ton	cwt.	lb.	
107	~~20~~ 19	166	
~~108~~	~~0~~	~~54~~	
− 79	11	86	
28	8	80	*Ans.*

ton	cwt.	lb.	
33	~~20~~ 19		
~~34~~	~~0~~	~~0~~ *	
− 12	3	105	
21	16	7	*Ans.*

* No helping figure should be allowed in cases like this. Give an exercise of at least 10 of these. Conclude with a period of revision practice on all steps.

SHORT MULTIPLICATION

Step 1. Short multiplication of cwt. and lb. with tons in answer.

Pay attention to the spacing of the sum. Underline working in the lb. column may be crushed if the lb. and cwt. are not well separated.

ton	cwt.	lb.	
	13	57	
		×9	
6	1	65	*Ans.*
6	4	4	
	117	112)513	
	2Ø)121	448	
	6 r 1	65 lb.	

Make at least 5 examples for class practice.

Step 2. Short multiplication of ton. cwt. lb.

ton	cwt.	lb.	
37	19	87	
		×11	
417	17	61	*Ans.*
10	8	8	
407	209	112)957	
417	2Ø)217	896	
	10 r 17	61 lb.	

Make at least 10 examples for class practice. Include *one or two only* with 0 cwt. in multiplicand.

LONG MULTIPLICATION

Step 1. Long multiplication of cwt. lb., with tons in answer.

The method of multiplying by 2 digits as taught in number *must* be followed, that is, multiplying by the unit figure first and the tens figure next. It is advisable to demonstrate and exercise

with multipliers under 20 at first. Then after an exercise of 10 sums, give an exercise with multipliers under 100.

ton	cwt.	lb.	
	15	96	
		×34	
26	19	16	*Ans.*
26	29	384	
	60	2880	
	450	29	
	$2\cancel{0})53\cancel{9}$	112)3264	
	26 r 19	224	
		1024	
		1008	
		16 lb.	

ton	cwt.	lb.	
	19	111	
		×87	
86	19	25	*Ans.*
86	86	777	
	133	8880	
	1520	86	
	$2\cancel{0})173\cancel{9}$	112)9657	
	86 r 19	896	
		697	
		672	
		25 lb.	

LONG MULTIPLICATION

Incidental Difficulties

These have not been called a separate step, as by this time most of the class should be able to deal with them. They are

given here so that the teacher may be aware that they exist and that some children may need help with them.

(*a*) Lb. multiplying to tons. (Include one or two with the general examples.)

ton	cwt.	lb.	
		109	
		× 98	
4	15	42	*Ans.*
	$2\cancel{0})9\cancel{5}$	872	
	4 r 15	9810	
		95	
		112)10682	
		1008	
		602	
		560	
		42 lb.	

(*b*) Tons. cwt. and lb. with 0 cwt. (Include one or two with the general examples.)

ton	cwt.	lb.	
35	0	84	
		× 46	
1611	14	56	*Ans.*
1	$2\cancel{0})3\cancel{4}$	504	
210	1 r 14	3360	
1400		34	
		112)3864	
		336	
		504	
		448	
		56 lb.	

LONG DIVISION

Step 1. Division, without remainder, of cwt. lb. with carrying. Divisor under 20.

cwt.	lb.	
1	12	*Ans.*
14)15	56	
14	112	
1	168	
×112	14	
112 lb.	28	
	28	
	—	

Give at least 10 in an exercise.

Step 2. Introduce tons. No remainders and carrying from both columns.

(*a*) Divisors less than 20.

ton	cwt.	lb.	
2	13	15	*Ans.*
14)37	3	98	
28	180	112	
9	183	210	
×20	14	14	
180 cwt.	43	70	
	42	70	
	1	—	
	×112		
	112 lb.		

Give an exercise of at least 10 sums like these.

(*b*) Divisors less than 100.

	ton	cwt.	lb.	
	9	18	23	*Ans.*
46)	455	17	50	
	414	820	1008	
	41	837	1058	
	×20	46	92	
	820 cwt.	377	138	
		368	138	
		9		
		×112		
		1008 lb.		

Give an exercise of at least 10 sums like these.

Step 3. Introduce 0 in dividend.

This ought not to give difficulties, but a blackboard demonstration should be given, done with the children's participation

(*a*) 0 lb. in dividend.

	ton	cwt.	lb.	
	2	13	7	*Ans.*
48)	127	7	0	
	96	620	336	
	31	627	336	
	×20	48	336	
	620 cwt.	147		
		144		
		3		
		×112		
		336 lb.		

(*b*) 0 cwt. in dividend.

```
         ton          cwt.         lb.
           6            17           7  Ans.
      ________________________________
      34)233             0          14
         204           580         224
         ___           ___         ___
          29           580         238
         ×20           34          238
        ____           ___         ___
         580 cwt.      240
                       238         ___
                       ___
                         2
                      ×112
                      ____
                       224 lb.
```

Give an exercise of 10 sums of Step 3.

Step 4. Practice with 3-digit divisors.

In weight, some work may be done with three-digit divisors without having unnecessarily large figures in the body of the sum. A certain amount of practice may be worthwhile and should be attempted. Revise, first, long division of pure number with this kind of divisor. When setting your own examples, keep your divisors below 250 and *never* have more than four figures of tons. The aim is to give *practice*.

```
          ton          cwt.         lb.
            2            15          24  Ans.
       ________________________________
      249)687             8          40
          498          3780        5936
          ___          ____        ____
          189          3788        5976
          ×20          249         498
         ____          ____        ____
         3780 cwt.     1298         996
                       1245         996
                       ____        ____
                         53
                       ×112        ____
                       ____
                        336
                       5600
                       ____
                       5936 lb.
```

The class will need at least 10 sums to practise by themselves. Teachers are advised to see that the lb. column of the answer is limited to two figures, and those preferably below 50.

They should make up the exercises by multiplying the answer they require by a figure suitable to each step. In this way remainders will be avoided, as they must be up to this point.

Step 5. Introduce remainders.

(*a*) Quantity in lb. column, remainder in lb. or cwt. lb.

(i)

ton	cwt.	lb.
13	13	11 r 14 lb. *Ans.*
151)2061	17	107
151	1960	1568
551	1977	1675
453	151	151
98	467	165
× 20	453	151
1960 cwt.	14	14 lb. remainder
	× 112	
	448	
	1120	
	1568 lb.	

(ii)

ton	cwt.	lb.
9	14	12 r 1 cwt. 4 lb. *Ans.*
117)1135	11	64
1053	1640	1456
82	1651	1520
× 20	117	117
1640 cwt.	481	350
	468	234
	13	116 lb. remainder
	× 112	
	336	
	1120	
	1456 lb.	

(*b*) 0 lb. in answer, remainder in lb. or cwt. lb.

	ton	cwt.	lb.
(i)	2	18	0 r 37 lb. *Ans.*
	54)156	12	37
	108	960	0
	48	972	37 lb. remainder
	×20	54	
	960 cwt.	432	
		432	

	ton	cwt.	lb.
(ii)	2	15	0 r 2 cwt. *Ans.*
	241)662	17	0
	482	3600	224
	180	3617	224 lb. = 2 cwt. remainder
	×20	241	
	3600 cwt.	1207	
		1205	
		2	
		×112	
		224 lb.	

The class should practise about 12 examples on this step—3 from each section.

DIVISION OF WEIGHT BY WEIGHT (*Optional*)

The class will be familiar with the division of length by length, and will have proved the accuracy of the method by practice. The rule is similar, namely, 'Reduce both weights to the lowest unit and divide.'

Introduction

Begin by revising—or recalling to the class the practical work they did at this stage of linear measure, and the rule which you

worked out with them as a result of what they discovered by the practical method. If the school has a pair of scales, do some more practical work—e.g. weigh a number of 8 oz. or 4 oz. weights (or sandbags) against a 5 lb. weight or sandbag, and find out how many of the former are needed to balance the latter. If you have used a 4 oz. weight and a 5 lb. weight, show the mechanical process on the blackboard as follows:

lb.		oz.
5	÷	4
×16		
4)80 oz.		
20 times		

Continue the practical work for some time, until the class are quite familiar with the process as applied to weight and are sure that the mechanical method is accurate. Weigh and work out mechanically.

	lb.	oz.		oz.
(i)	3	8	÷	4
and (ii)	7	5	÷	9

according to the equipment you have. The method of writing down is the same as in Linear Measure. Then go on to the mechanical work. The steps given are those which are considered most practical.

Step 1. (*a*) lb. oz. ÷ oz.; and (*b*) lb. oz. ÷ lb. oz.

	lb.	oz.		lb.	oz.
(*a*)	34	2	÷		13
	×16	544			
	204	42 times *Ans.*			
	340	13)546			
	544 oz.	52			
		26			
		26			
		..			

	lb.	oz.		lb.	oz.
(*b*)	67	14	÷	11	5
	×16	1072		×16	176
	402	1086		176 oz.	181
	670				
	1072 oz.				

6 times *Ans.*

181)1086
1086
....

The working out of division is done separately, below the reduction.

Step 2. (*a*) cwt. lb. ÷ lb. and (*b*) cwt. lb. ÷ cwt. lb.

	cwt.	lb.		lb.
(*a*)	17	58	÷	109
	×112	1904		
	784	1962		
	1120			
	1904 lb.			

18 times *Ans.*

109)1962
109
872
872
...

	cwt.	lb.		cwt.	lb.
(*b*)	14	105	÷	2	15
	×112	1568		×112	224
	448	1673		224 lb.	239
	1120				
	1568 lb.				

7 times *Ans.*

239)1673
1673
....

FRACTIONS

MULTIPLICATION

Step 1. Revision of all ground covered to date, i.e. the addition of up to three items, together with subtraction of fractions and mixed whole units and fractions with initial denominators (excluding 7 and 11) up to 12.

This step will probably take only about 2 lessons, but during this time you should ensure that every child in the class understands and can cope with all of the steps involved. If, for instance, it is discovered that 3 or 4 children still cannot cope with, say, the conversion of whole numbers to improper fractions, they should be taken aside and taught the step again, reference being made to the appropriate sections of Class 4 and/or Class 3 work in these notes.

Step 2. Introduction to the multiplication of fractions.

(*a*) Revision of the meaning of fractions as dealt with in Step 4 of Term 3 of Class 3.

(*b*) Thus, all of the children having been reminded quite clearly, that, for instance, $\frac{1}{3}=1\div3$, and also that $\frac{3}{1}=3\div1$, and so on,

the teacher then develops the idea as follows:

(i) We have seen that

$$\tfrac{5}{8}=5\div8$$

$\therefore$ (ii) If we multiply $\frac{5}{8}$ by, say 1, to start with, we get

$$1\times\tfrac{5}{8}=1\times5\div8$$

Now $1\times5=5$, and as we have already revised above

$$5\div8=\tfrac{5}{8}$$

Thus $\quad 1\times\frac{5}{8}=1\times5\div8=\frac{5}{8}$

(iii) The next step is to see what happens if we multiply by 2, rather than 1.

$$2\times\tfrac{5}{8}=2\times5\div8$$

Now $\quad 2\times5=10, \quad$ and $\quad 10\div8=\frac{10}{8}$

So we see that

$$2 \times \frac{5}{8} = 2 \times 5 \div 8 = \frac{\overset{5}{\cancel{10}}}{\underset{4}{\cancel{8}}} = 1\frac{1}{4} \quad \textit{Ans.}$$

or again $\quad 3 \times \frac{5}{8} = 3 \times 5 \div 8 = \frac{15}{8} = 1\frac{7}{8}$ *Ans.*

(iv) The next step is to see what happens if we divide first rather than multiply. Thus an example can be worked twice in different ways on the blackboard as follows :

$$4 \times \frac{5}{8} = 4 \times 5 \div 8 = \frac{\overset{5}{\cancel{20}}}{\underset{2}{\cancel{8}}} = 2\frac{1}{2} \quad \textit{Ans.}$$

$$4 \times \frac{5}{8} = 4 \div 8 \times 5 = \frac{1}{2} \times 5 = \frac{5}{2} = 2\frac{1}{2} \quad \textit{Ans.}$$

A second example should then be dealt with in the same way :

$$2 \times \frac{3}{4} = 2 \times 3 \div 4 = \frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}} = 1\frac{1}{2} \quad \textit{Ans.}$$

$$2 \times \frac{3}{4} = 2 \div 4 \times 3 = \frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}} \times 3 = \frac{3}{2} = 1\frac{1}{2} \quad \textit{Ans.}$$

Two or three more examples can be worked if necessary, until it is quite clear that the same correct result is obtained every time, no matter whether we multiply first or divide first.

(*c*) Thus the children see the rule that :

It does not matter whether the multiplication or the division is done first *as long as each part* of the fraction *is treated correctly*, that is, the top (numerator) is a multiplying and the bottom (denominator) is a dividing factor.

(*d*) The working of simple multiplications of whole units by

fractions as follows after the first one has been worked as the demonstration :

$5 \times \frac{7}{8} = \frac{35}{8} = 4\frac{3}{8}$ *Ans.*

$4 \times \frac{3}{5}$; $3 \times \frac{7}{10}$; $4 \times \frac{5}{6}$; $5 \times \frac{2}{3}$

$8 \times \frac{5}{9}$; $4 \times \frac{2}{3}$; $7 \times \frac{5}{12}$; $6 \times \frac{7}{8}$; $11 \times \frac{1}{6}$

$9 \times \frac{3}{4}$; $8 \times \frac{2}{5}$; $4 \times \frac{1}{6}$; $6 \times \frac{3}{10}$; $11 \times \frac{1}{4}$

$7 \times \frac{3}{8}$; $4 \times \frac{7}{9}$; $11 \times \frac{1}{9}$; $7 \times \frac{11}{12}$; $10 \times \frac{1}{8}$; $11 \times \frac{9}{10}$

(*e*) The use of the word 'of' in fractions. Ask the class what is $\frac{1}{2}$ of 4 and write on the blackboard $\frac{1}{2}$ of $4=2$.

Repeat this with other examples so the blackboard reads

$\frac{1}{2}$ of $4=2$

$\frac{1}{2}$ of $7=3\frac{1}{2}$

$\frac{1}{3}$ of $9=3$

$\frac{1}{4}$ of $12=3$

but ending with an example which most, if not all, children will fail to answer

$\frac{1}{8}$ of $18=$

Point out to the class that we can solve some of these sums mentally but as is seen we will meet others that we cannot do in that way, and need therefore to find some other way.

Draw the attention of the class to the first blackboard example and say you are going to do it another way.

Write on the blackboard

$\frac{1}{2} \times 4$

and complete the sum as follows

$= \frac{4}{2} = 2$

Draw the attention of the class again to the fact that substituting '$\times$' for 'of' produces the same answer. Prove it again with another of the blackboard sums, and then proceed to the last unsolved example, working it on the blackboard with the children.

$\frac{1}{8}$ of 18

$= \frac{1}{8} \times 18$

$= \frac{18}{8} = 2\frac{1}{4}$

Give an exercise of 10 similar sums.

Step 3. The multiplication of fractions by fractions.

(*a*) (i) The teacher explains that as we have learnt above (Step 2) to multiply whole numbers by fractions, it is just as easy to multiply fractions by fractions.

The only difference is that we now have 2 bottoms (denominators) to multiply together, as well as 2 tops (numerators).

Thus whereas before we had :

$$3 \times \frac{5}{8}\left(=\frac{3 \times 5}{8}\right)=\frac{15}{8}=1\tfrac{7}{8}$$

Now we might have

$$\frac{3}{4} \times \frac{5}{8}\left(=\frac{3 \times 5}{4 \times 8}\right)=\frac{15}{32}$$

Thus (ii) We see that the multiplying sign applies not just to the numerator or just to the denominator, but to both. This having been clearly established, the children can then go on to work the following after the teacher has demonstrated the first two :

$$\frac{7}{8} \times \frac{5}{6}=\frac{7 \times 5}{8 \times 6}=\frac{35}{48} \quad \textit{Ans.}$$

$$\frac{4}{5} \times \frac{1}{6}=\frac{4 \times 1}{5 \times 6}=\frac{\overset{2}{\cancel{4}}}{\underset{15}{\cancel{30}}}=\frac{2}{15} \quad \textit{Ans.}$$

$\frac{3}{8} \times \frac{1}{2}$; $\frac{5}{8} \times \frac{3}{4}$; $\frac{7}{12} \times \frac{1}{3}$; $\frac{4}{9} \times \frac{2}{3}$

$\frac{7}{10} \times \frac{3}{8}$; $\frac{5}{6} \times \frac{5}{8}$; $\frac{3}{8} \times \frac{5}{12}$; $\frac{11}{12} \times \frac{1}{4}$

$\frac{2}{3} \times \frac{7}{9}$; $\frac{1}{2} \times \frac{3}{10}$; $\frac{4}{5} \times \frac{2}{9}$; $\frac{7}{12} \times \frac{1}{10}$

(*b*) Now that the principle of multiplying both numerators is clearly understood the next step of cancelling *before* multiplication is demonstrated as follows :

$$\frac{3}{8} \times \frac{4}{9}=\frac{3 \times 4}{8 \times 9}=\frac{\overset{\cancel{6}\;1}{\cancel{12}}}{\underset{\cancel{36}\;6}{\cancel{72}}}=\frac{1}{6} \quad \textit{Ans.}$$

can be done as follows :

$$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6} \textit{ Ans.}$$

Thus it seems that where there is a *multiplication* sign between two fractions, we can cancel first between numerator and any denominator without having to wait until we get to the answer.

Further examples can be demonstrated if necessary—e.g.

$$\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{12}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{10}}} = \frac{1}{8} \textit{ Ans.},$$

after which the children can proceed with their own examples :

$\frac{3}{8} \times \frac{2}{3}$; $\frac{11}{12} \times \frac{3}{4}$, and so on.

The teacher should make up at least 30 more examples for the children to practise. (*Note:* All fractions to be less than one whole unit and within the denominators known.)

(*c*) Introduction of whole numbers in multiplying.

(i) It is advisable here to revise briefly the rules for addition and subtraction of fractions as summarised in Step 9(*a*) of Class 4.

Then emphasise that as with addition and subtraction so with multiplication (and division), the *first step is always* to change whole numbers with fractions into improper fractions before commencing any other steps of the working.

(ii) Demonstrations on the blackboard of :

(*a*) $2\frac{1}{2} \times \frac{7}{8} = \frac{5}{2} \times \frac{7}{8} = \frac{35}{16} = 2\frac{3}{16}$ *Ans.*

(*b*)

$$\frac{5}{8} \times 2\frac{2}{3} = \frac{5}{\underset{1}{\cancel{8}}} \times \frac{\overset{1}{\cancel{8}}}{3} = \frac{5}{3} = 1\frac{2}{3} \textit{ Ans.}$$

(*c*)

$$1\tfrac{1}{2}\times 1\tfrac{4}{9}=\frac{\overset{1}{\cancel{3}}}{2}\times\frac{13}{\underset{3}{\cancel{9}}}=\frac{13}{6}=2\tfrac{1}{6}\ \textit{Ans.}$$

(iii) Children to do 30 examples themselves, as set by the teacher. *Rules:* Examples should be of types (*a*), (*b*) and (*c*) of (ii) above mixed, whole numbers not to exceed 3, and working with the fractional parts learnt.

(*d*) Introduction of 3 items in multiplying.

(i) Explain that this is on just the same principles as for two items, but perhaps a little more complicated, then demonstrate:

(*a*)

$$\frac{1}{2}\times\frac{7}{8}\times\frac{4}{5}=\frac{1\times 7\times \overset{1}{\cancel{4}}}{2\times \underset{2}{\cancel{8}}\times 5}=\frac{7}{20}\ \textit{Ans.}$$

(*b*)

$$1\tfrac{1}{2}\times\frac{3}{4}\times\frac{5}{12}=\frac{\overset{1}{\cancel{3}}}{2}\times\frac{3\times 5}{4\times \underset{4}{\cancel{12}}}=\frac{15}{32}\ \textit{Ans.}$$

(*c*)

$$1\tfrac{1}{3}\times\frac{7}{8}\times 3\tfrac{3}{4}$$

$$=\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}\times\frac{7}{8}\times\frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{4}}}=\frac{35}{8}=4\tfrac{3}{8}\ \textit{Ans.}$$

(*d*)

$$\frac{5}{8}\times 3\times 1\tfrac{5}{9}$$

$$=\frac{5}{\underset{4}{\cancel{8}}}\times\frac{\overset{1}{\cancel{3}}}{1}\times\frac{\overset{7}{\cancel{14}}}{\underset{3}{\cancel{9}}}=\frac{35}{12}=2\tfrac{11}{12}\ \textit{Ans.}$$

(ii) Teacher to set 10 each of (*a*), (*b*), (*c*) and (*d*) above to the children. *Rules:* Whole numbers not to exceed 3, and not to appear in more than two of the three items. Work, as usual, within the fractional parts learnt.

DIVISION

Step 1. An introduction to the division of fractions. Put the following sums on the left side of the blackboard :

$$6 \div 2 =$$
$$10 \div 2 =$$
$$16 \div 2 =$$
$$24 \div 2 =$$
$$11 \div 2 =$$

Obtain the answers from the class and write them on the blackboard.

Ask the class to give another way of reading or saying the first sum and elicit ' a half of 6 '. Do the same with all examples and write them on the side so the blackboard reads :

$6 \div 2 = 3$	$\frac{1}{2}$ of $6 = 3$
$10 \div 2 = 5$	$\frac{1}{2}$ of $10 = 5$
$16 \div 2 = 8$	$\frac{1}{2}$ of $16 = 8$
$24 \div 2 = 12$	$\frac{1}{2}$ of $24 = 12$
$11 \div 2 = 5\frac{1}{2}$	$\frac{1}{2}$ of $11 = 5\frac{1}{2}$

Question the class on the meaning of the word ' of ', eliciting the answer that it means ' Multiply '. Replace the word ' of ' by ' $\times$ ' on the blackboard.

Point out that $\frac{1}{2} \times 6$ can be written $6 \times \frac{1}{2}$ and write this on the blackboard so that the blackboard reads :

$6 \div 2 = 3$	$\frac{1}{2} \times 6 = 3$	$6 \times \frac{1}{2} = 3$
$10 \div 2 = 5$	$\frac{1}{2} \times 10 = 5$	$10 \times \frac{1}{2} = 5$
$16 \div 2 = 8$	$\frac{1}{2} \times 16 = 8$	$16 \times \frac{1}{2} = 8$
$24 \div 2 = 12$	$\frac{1}{2} \times 24 = 12$	$24 \times \frac{1}{2} = 12$
$11 \div 2 = 5\frac{1}{2}$	$\frac{1}{2} \times 11 = 5\frac{1}{2}$	$11 \times \frac{1}{2} = 5\frac{1}{2}$

Ask the class if it finds an interesting fact in a comparison of the sums on the left with those on the right, and lead the class to note that the divisor, or second number of the sums on the left, has been turned around in the sums on the right and the division sign has been changed to the multiplication sign.

Step 2. The division of whole numbers by fractions :

Put on the blackboard

$$9 \div \tfrac{1}{2} =$$

and obtain the answer if possible. Most, if not all, children will probably fail at this stage.

Point out that when we say $9 \div 3 = 3$, we mean 'how many 3s in 9?' Thus the sum on the blackboard means 'how many $\frac{1}{2}$s in 9?' The class will now give the answer, 18, which is written on the blackboard to complete the sum.

Now say to the class

'Let us change the division sign to the multiplication sign, turn the fraction $\frac{1}{2}$ round as we learnt to do earlier.' The blackboard reads :

$$9 \div \tfrac{1}{2}$$
$$= 9 \times \tfrac{2}{1} = 18$$

The teacher should demonstrate a second example,

$$5 \div \tfrac{2}{3}$$
$$= 5 \times \tfrac{3}{2}$$
$$= \tfrac{15}{2} = 7\tfrac{1}{2}$$

Having made sure the children have *all* clearly followed these examples, the teacher can then introduce the 'turn upside down and multiply' rule for dividing fractions.

The children can then do the following exercise :

$12 \div \frac{1}{2}$;	$3 \div \frac{1}{5}$;	$5 \div \frac{1}{3}$;	$4 \div \frac{1}{4}$;
$12 \div \frac{2}{3}$;	$6 \div \frac{3}{4}$;	$4 \div \frac{2}{5}$;	$7 \div \frac{3}{8}$;
$8 \div \frac{1}{10}$;	$2 \div \frac{5}{12}$;	$5 \div \frac{4}{9}$;	$3 \div \frac{7}{10}$;
$4 \div \frac{7}{8}$;	$5 \div \frac{1}{6}$;	$2 \div \frac{3}{5}$;	$4 \div \frac{3}{10}$.

Step 3. (*a*) The division of fractions by fractions.

Put on the blackboard

$$\tfrac{1}{2} \div \tfrac{1}{2} =$$

It is probable that many, if not all, children will be able to answer this sum. In any case, remind the class that the sum means 'how many $\frac{1}{2}$s in $\frac{1}{2}$?'

Write the answer on the blackboard.

Ask a child to do the sum by the rule of ‘ turn upside down and multiply ’, so the blackboard reads :

$$
\begin{aligned}
&\tfrac{1}{2} \div \tfrac{1}{2} \\
&= \tfrac{1}{2} \times \tfrac{2}{1} \\
&= \tfrac{2}{2} = 1
\end{aligned}
$$

Demonstrate a second example $\frac{3}{4} \div \frac{1}{2}$, and a third $\frac{3}{4} \div \frac{2}{3}$.

At this point it is advisable to emphasise that cancelling is *never* done with a division sign.

Thus on the third example we *cannot* say

$$
\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} \div \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} = \frac{1}{2}
$$

as is proved by the correct way which gives the answer $1\frac{1}{8}$.

$$
\begin{aligned}
&\tfrac{3}{4} \div \tfrac{2}{3} \\
&= \tfrac{3}{4} \times \tfrac{3}{2} \\
&= \tfrac{9}{8} = 1\tfrac{1}{8}
\end{aligned}
$$

Cancellation must only be done with the multiplication sign.

The class proceeds to an exercise of about 12 simple 2-item division sums, with no whole numbers.

The introduction of whole numbers in division.

The teacher explains that the rule is exactly the same as in addition, subtraction and multiplication ; always convert the proper fractions to improper fractions before commencing any other working.

Put on the blackboard and work with the class :

$$
\begin{aligned}
&1\tfrac{1}{2} \div \tfrac{2}{5} \\
&= \tfrac{3}{2} \div \tfrac{2}{5} \\
&= \tfrac{3}{2} \times \tfrac{5}{2} \\
&= \tfrac{15}{4} = 3\tfrac{3}{4}
\end{aligned}
$$

Demonstrate with the class $\frac{7}{8} \div 1\frac{1}{2}$ and $5\frac{1}{2} \div 2\frac{1}{4}$.

The children work about 20 mixed examples such as:

$$4\div\tfrac{2}{3}\,;\qquad \tfrac{7}{8}\div\tfrac{2}{3}\,;\qquad 2\tfrac{3}{4}\div1\tfrac{3}{8}\,;$$
$$3\tfrac{1}{2}\div2\,;\qquad 5\div1\tfrac{1}{2}\,;\qquad 1\tfrac{2}{5}\div\tfrac{3}{10}.$$

Rule : The whole numbers not to exceed 6 and only known denominators to be used.

(*b*) Double divisions (i.e. 3 items).

(i) Explain that (as in multiplication) this is just the same as for 2 items, except perhaps a little more complicated, for we now have to turn upside down both the *second* and *third* items.

(ii) Blackboard demonstrations :

(*a*)

$$\frac{1}{2}\div\frac{1}{3}\div\frac{7}{8}$$
$$=\frac{1}{\underset{1}{\cancel{2}}}\times\frac{3}{1}\times\frac{\overset{4}{\cancel{8}}}{7}=\frac{12}{7}=1\tfrac{5}{7}\ \textit{Ans.}$$

(*b*)

$$1\tfrac{1}{2}\div\tfrac{1}{3}\div2\tfrac{1}{3}$$
$$=\tfrac{3}{2}\div\tfrac{1}{3}\div\tfrac{7}{3}=\tfrac{3}{2}\times\tfrac{3}{1}\times\tfrac{3}{7}=\tfrac{27}{14}=1\tfrac{13}{14}\ \textit{Ans.}$$

(*c*)

$$2\tfrac{1}{4}\div3\div\tfrac{5}{8}$$
$$=\frac{9}{4}\div\frac{3}{1}\div\frac{5}{8}=\frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}}\times\frac{1}{\underset{1}{\cancel{3}}}\times\frac{\overset{2}{\cancel{8}}}{5}=\frac{6}{5}=1\tfrac{1}{5}\ \textit{Ans.}$$

(iii) Teacher to set 10 examples for the class (mixed (*a*), (*b*) and (*c*) above). *Rules:* Never more than 2 items with whole units, neither of which should exceed 6. Do not have whole units in both second *and* third items. Keep to learnt fractional parts.

(*c*) To conclude, the rules and order of working in the division of fractions may be summarised as follows :

(i) Change all items to improper fractions from whole units.

(ii) Any item *preceded* by a division sign should be inverted and the sign changed from ÷ to × at the same time

(iii) Any appropriate cancelling between numerators and denominators may now be done to save working with unduly high figures. (N.B. Some children may try to cancel, say, two denominators against each other : watch for this and explain this rule wherever the fault occurs.)

(iv) Multiply all numerators together and all denominators together.

(v) Convert to whole units in answer if this step is required.

Note on the multiplication and division of fractions

It will be noticed that in the Class 5 work covered in Steps 1 to 6 no mention has been made of the visual aids used a great deal in the two earlier classes.

This omission is not so much because the children can now immediately dispense with visual aids altogether (although they should gradually be arriving at that point) but rather because by this stage the variations between one child and another and between one class and another will be such that while some can cover almost everything in the above steps purely theoretically with just an occasional practical reminder if they are well taught ; others will need a thorough revision complete with visual aids and practise with these aids at the beginning, and a continuing use of these aids in the later stages.

(ii) If the latter is the case, the teacher will have to proceed more slowly, doing plenty of practical work in groups at these stages. Every effort should be made to train the children to think these things out abstractly as well, though, as by the time the more advanced steps are reached, any practical work is not only very complicated but also very time-consuming.

(iii) It is recommended that from this point onwards no attempt should be made to go into further practical demonstrations (except in revision of an earlier step accompanied by revision of its own earlier practical work) but new ideas should be introduced as simply as possible in small steps, but purely mentally. The blackboard figures and symbols should now be understood enough to serve as their own visual aids, and for those children for whom this is not so, it is doubtful if further complicated practical work will help. The teacher should rather go back with his small group of slow children over the basic work (practical included) done in previous classes and attempt to discover and eliminate the weak points in the child's understanding at that level.

MIXED SYMBOL FRACTIONS

Step 1. Mixed addition and subtraction.

(*a*) Three items of addition have already been introduced in Step 10 of Class 4. All that is required now is to vary one of the symbols and instruct the children *always to do the addition before subtracting* (if they are in any doubt).

(*b*) Blackboard demonstration by the teacher is as follows :

(i) $1\frac{1}{2}+\frac{7}{8}-1\frac{5}{6}$

$$=\frac{3}{2}+\frac{7}{8}-\frac{11}{6}=\frac{36+21-44}{24}=\frac{57-44}{24}=\frac{13}{24} \textit{ Ans.}$$

(ii) $1\frac{5}{6}-2\frac{1}{4}+1\frac{2}{3}$

$$=\frac{11}{6}-\frac{9}{4}+\frac{5}{3}=\frac{22+20-27}{12}=\frac{42-27}{12}=\frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{12}}}=1\frac{1}{4} \textit{ Ans.}$$

(*c*) Teacher to make up 15 of type (i) above, and then follow this with 15 of type (ii). *Rules :* Known fractional parts, whole units not to exceed 3, and answers always to be positive.

(*d*) (i) Introduce 4 items. The method is exactly the same, the only difference being that the sums are slightly more complicated.

(ii) Blackboard demonstrations :

$$1\tfrac{1}{4}+\tfrac{5}{8}+\tfrac{2}{3}-2\tfrac{1}{6}$$

$$=\frac{5}{4}+\frac{5}{8}+\frac{2}{3}-\frac{13}{6}$$

$$=\frac{30+15+16-52}{24}=\frac{61-52}{24}=\frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{24}}}=\frac{3}{8}\ \textit{Ans.}$$

(iii) Children to do the following examples :

$1\tfrac{3}{4}+1\tfrac{1}{6}+\tfrac{2}{3}-2\tfrac{1}{3}$; $\tfrac{7}{10}+\tfrac{5}{6}+\tfrac{4}{5}-1\tfrac{1}{3}$

$2\tfrac{1}{2}+1\tfrac{5}{9}-2\tfrac{5}{6}+1\tfrac{1}{3}$; $\tfrac{11}{12}+1\tfrac{1}{4}-2\tfrac{1}{2}+1\tfrac{1}{3}$

$\tfrac{5}{6}-2\tfrac{7}{10}+1\tfrac{2}{3}+\tfrac{4}{5}$; $\tfrac{7}{8}-3\tfrac{1}{2}+1\tfrac{9}{10}+1\tfrac{3}{4}$

(iv) (*a*) Now the further complication of 2 subtractions can be introduced, first by taking each away in separate stages.

$$2\tfrac{1}{4}+1\tfrac{1}{3}-1\tfrac{1}{2}-1\tfrac{5}{6}$$

$$=\frac{9}{4}+\frac{4}{3}-\frac{3}{2}-\frac{11}{6}$$

$$=\frac{27+16-18-22}{12}=\frac{43-18-22}{12}=\frac{25-22}{12}$$

$$=\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{12}}}=\frac{1}{4}\ \textit{Ans.}$$

This can then be built up to the realisation that we get the same result more quickly if we *add* the 2 (or 3) subtraction items together :

(*b*) $2\tfrac{1}{4}+1\tfrac{1}{3}-1\tfrac{1}{2}-1\tfrac{5}{6}$

$$=\frac{9}{4}+\frac{4}{3}-\frac{3}{2}-\frac{11}{6}=\frac{27+16-18-22}{12}$$

$$=\frac{27+16-(18+22)}{12}=\frac{43-40}{12}=\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{12}}}=\frac{1}{4}\ \textit{Ans.}$$

Whether this appears to be clear or not, two more examples should be worked as follows (the bracketed step above now being done only verbally):

(*c*) $1\frac{7}{8}-2\frac{5}{6}+2\frac{3}{4}-1\frac{1}{3}$

$$=\frac{15}{8}-\frac{17}{6}+\frac{11}{4}-\frac{4}{3}$$

$$=\frac{45+66-68-32}{24}=\frac{111-100}{24}=\frac{11}{24}\ \textit{Ans.}$$

(*d*) $\frac{9}{10}-1\frac{1}{3}-2\frac{1}{6}+2\frac{4}{5}$

$$=\frac{9}{10}-\frac{4}{3}-\frac{13}{6}+\frac{14}{5}$$

$$=\frac{27+84-40-65}{30}=\frac{111-105}{30}=\frac{\overset{1}{\cancel{6}}}{\underset{5}{\cancel{30}}}=\frac{1}{5}\ \textit{Ans.}$$

(v) Teacher to set children at least 5 examples each of (*a*), (*b*) and (*c*) above in turn. *Rules:* Exactly as in Step 8(*c*) above.

(vi) Introduction of 3 subtractions. Demonstration as follows:

$$3\tfrac{3}{4}-1\tfrac{1}{3}-1\tfrac{5}{12}-\tfrac{1}{6}$$

$$=\frac{15}{4}-\frac{4}{3}-\frac{17}{12}-\frac{1}{6}$$

$$=\frac{45-16-17-2}{12}$$

$$=\frac{45-35}{12}=\frac{\overset{5}{\cancel{10}}}{\underset{6}{\cancel{12}}}=\frac{5}{6}\ \textit{Ans.}$$

(vii) Children to do 5 examples of (vi) above. *Rules:* Initial number not in excess of 9, others not in excess of 3, and all answers to be positive.

(viii) Teacher to set 12 examples covering (*a*), (*b*) and (*c*) above. *Rules:* As above and order to be mixed up.

Step 2. Mixed multiplication and division, including ‘ of ’.

(*a*) This is quite simple, as the children already do division by multiplying after turning the appropriate term upside down. The only point for caution is to ensure that all the children understand quite clearly that the division symbol applies to the number which comes immediately *after* it.

(*b*) Blackboard examples demonstrated by the teacher :

(i)

$$\frac{3}{4}\times\frac{5}{6}\div\frac{2}{3}=\frac{\overset{1}{\cancel{3}}}{4}\times\frac{5}{\underset{2}{\cancel{6}}}\times\frac{3}{2}=\frac{15}{16}\ \textit{Ans.}$$

(ii)

$$\frac{4}{5}\div\frac{7}{10}\times\frac{1}{2}=\frac{4}{\underset{1}{\cancel{5}}}\times\frac{\overset{\cancel{2}}{\cancel{10}}}{7}\times\frac{\overset{1}{1}}{\underset{1}{\cancel{2}}}=\frac{4}{7}\ \textit{Ans.}$$

(*c*) Children work 10 each of (*a*) and (*b*) above, as set by the teacher.

(*d*) Introduce ‘ of ’ and explain that it means exactly the same as $\times$, so at the first opportunity replace by this latter symbol. Demonstrate as follows :

Prove ‘ of ’ means $\times$ by example : $\frac{1}{4}$ of $4=1$ etc.

(i)

$$\frac{11}{12}\text{ of }\frac{3}{4}\times\frac{2}{3}=\frac{11}{\underset{4}{\cancel{12}}}\times\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}}\times\frac{\overset{1}{\cancel{2}}}{3}=\frac{11}{24}\ \textit{Ans.}$$

(ii)

$$\frac{5}{6}\text{ of }\frac{1}{2}\div\frac{2}{3}=\frac{5}{\underset{2}{\cancel{6}}}\times\frac{1}{2}\times\frac{\overset{1}{\cancel{3}}}{2}=\frac{5}{8}\ \textit{Ans.}$$

(iii)

$$\frac{3}{4}\div\frac{5}{12}\text{ of }\frac{5}{9}=\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}}\times\frac{\overset{\cancel{3}\,1}{\cancel{12}}}{\underset{1}{\cancel{5}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{\cancel{3}\,1}{\cancel{9}}}=\frac{1}{1}=1\ \textit{Ans.}$$

(iv)

$$\frac{3}{8}\times\frac{5}{6}\text{ of }\frac{2}{3}=\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}}\times\frac{5}{6}\times\frac{\overset{1}{2}}{\underset{1}{\cancel{3}}}=\frac{5}{24}\quad \textit{Ans.}$$

Teacher to set at least 20 examples, being 5 each of (i), (ii), (iii) and (iv) above for the children to do. Note that here we use vulgar fractions only and not whole numbers.

(*f*) Introduction of 4 items and improper fractions. This does not introduce anything new, but merely reintroduces complexities which have been omitted while the new points were being grasped. Demonstrate as follows:

(i) $\frac{11}{12}$ of $1\frac{1}{2}\times\frac{3}{4}\div\frac{5}{6}$

$$=\frac{11}{\underset{2}{\cancel{12}}}\times\frac{3}{2}\times\frac{3}{4}\times\frac{\overset{1}{\cancel{6}}}{5}=\frac{99}{80}=1\tfrac{19}{80}\quad \textit{Ans.}$$

(ii) $2\div\frac{1}{3}$ of $\frac{7}{8}\times\frac{3}{4}$

$$=\frac{\overset{1}{\cancel{2}}}{1}\times\frac{3}{1}\times\frac{7}{8}\times\frac{3}{\underset{2}{\cancel{4}}}=\frac{63}{16}=3\tfrac{15}{16}\quad \textit{Ans.}$$

(*g*) Teacher to set at least 20 examples of the type indicated in (*f*) above.

Rules: No whole units in excess of 2 and then not in more than 2 items. Keep to the fractional parts known.

Step 3. Mixed symbol fractions up to 4 items.

(*a*) Before this can be done, it is necessary to explain to the children the order of working. This is:

(i) As we have already seen, 'of' and division must be dealt with before multiplication, because both of them actually change into multiplications.

(ii) Multiplications (i.e. including ÷ and 'of') are always completed before any addition or subtraction is started.

(iii) When in doubt, addition comes before subtraction.

(*b*) Demonstration of these principles :

(i) $\frac{2}{3} \times 1\frac{1}{4} + \frac{5}{6}$

$$=\frac{\overset{1}{\cancel{2}}}{3} \times \frac{5}{\underset{2}{\cancel{4}}} + \frac{5}{6} = \frac{5}{6} + \frac{5}{6} = \frac{5+5}{6} = \frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{6}}} = 1\frac{2}{3}\ \textit{Ans.}$$

Remember we can only cancel across × signs, *not* across any of ÷, + or −.

(ii) $\frac{1}{2} \times 2\frac{1}{2} - \frac{3}{4}$

$$=\frac{1}{2} \times \frac{5}{2} - \frac{3}{4} = \frac{5}{4} - \frac{3}{4} = \frac{5-3}{4} = \frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}} = \frac{1}{2}\ \textit{Ans.}$$

(iii) $1\frac{1}{4} + \frac{2}{3} \times \frac{3}{4}$

$$=\frac{5}{4} + \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{5}{4} + \frac{1}{2} = \frac{5+2}{4} = \frac{7}{4} = 1\frac{3}{4}\ \textit{Ans.}$$

(iv) $2\frac{5}{6} - 1\frac{3}{4} \times \frac{4}{5}$

$$=\frac{17}{6} - \frac{7}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{5} = \frac{17}{6} - \frac{7}{5} = \frac{85-42}{30} = \frac{43}{30} = 1\frac{13}{30}\ \textit{Ans.}$$

(*c*) The teacher now sets at least 5 examples of each type, (i), (ii), (iii) and (iv) in (*b*) above, for the children to do.

Rules: Fractional parts known, no whole units above 3, and ensure that the answers are positive in (ii) and (iv).

(*d*) The same steps, (*b*) and (*c*) above, but this time with 4 items. Demonstrations as follows :

(i) $1\frac{1}{2} + 2\frac{1}{3} \times \frac{4}{5} - \frac{5}{6}$

$$=\frac{3}{2} + \frac{7}{3} \times \frac{4}{5} - \frac{5}{6} = \frac{3}{2} + \frac{28}{15} - \frac{5}{6}$$

$$=\frac{45+56-25}{30} = \frac{101-25}{30} = \frac{\overset{38}{\cancel{76}}}{\underset{15}{\cancel{30}}} = 2\frac{8}{15}\ \textit{Ans.}$$

(ii) $2\times\frac{1}{4}-1\frac{1}{3}+\frac{9}{10}$

$$=\frac{2}{1}\times\frac{1}{4}-\frac{4}{3}+\frac{9}{10}=\frac{2}{4}-\frac{4}{3}+\frac{9}{10}$$

$$=\frac{30+54-80}{60}=\frac{84-80}{60}=\frac{\overset{1}{\cancel{4}}}{\underset{15}{\cancel{60}}}=\frac{1}{15}\ \textit{Ans.}$$

(iii) $1\frac{1}{4}-2\frac{3}{8}+1\frac{2}{3}\times\frac{9}{10}$

$$=\frac{5}{4}-\frac{19}{8}+\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}}=\frac{5}{4}-\frac{19}{8}+\frac{3}{2}$$

$$=\frac{10+12-19}{8}=\frac{22-19}{8}=\frac{3}{8}\ \textit{Ans.}$$

(*e*) The working by the children of at least 5 examples of each type, (i), (ii) and (iii) above. Teacher should set examples with rules as in (*c*) above, i.e. no whole units in excess of 3, fractional parts known, and ensure positive answers.

(*f*) Introduction of ' of ' and (÷) complications.

(i) Remind the children that, as we have already seen, ÷ and ' of ' are really treated in exactly the same way as multiplication once they have been converted to ×, so that this work is really the same as in (*d*) and (*e*) above.

(ii) Demonstrate :

(*a*) $\frac{7}{8}$ of $1\frac{2}{3}-\frac{2}{5}\div\frac{3}{4}$

$$=\frac{7}{8}\times\frac{5}{3}-\frac{2}{5}\times\frac{4}{3}=\frac{35}{24}-\frac{8}{15}$$

$$=\frac{175-64}{120}=\frac{\overset{37}{\cancel{111}}}{\underset{40}{\cancel{120}}}=\frac{37}{40}\ \textit{Ans.}$$

(*b*) $1\frac{1}{4}+\frac{2}{3}\times\frac{1}{8}\div\frac{5}{6}$

$$=\frac{5}{4}+\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{1}{\underset{\cancel{4}\,2}{\cancel{8}}}\times\frac{\overset{\cancel{2}\,1}{\cancel{6}}}{5}=\frac{5}{4}+\frac{1}{10}=\frac{25+2}{20}=\frac{27}{20}=1\frac{7}{20}\ \textit{Ans.}$$

(*g*) Teacher to set at least 20 examples for the children to do, incorporating all five symbols in mixed orders.

Rules: (i) Maximum of 4 items; (ii) only known fractional parts up to 8 to be used, i.e. whole units and denominators of 2, 3, 4, 5, 6 and 8; (iii) whole units not to exceed 3 in any item; (iv) all answers to be positive, not negative.

Revision. (*a*) Revision of the four rules in fractions and a clear statement of the overall working order. This would in fact incorporate a complete revision of all work done to date in fractions throughout the school.

(*b*) The main points can be summarised in working order as follows:

(i) Always change all fractions with whole numbers to improper fractions throughout in all items;

(ii) Invert dividing fractions and change their preceding signs from $\div$ to $\times$;

(iii) Change 'of' to $\times$;

(iv) Cancel (across $\times$ signs only);

(v) Complete the working of all multiplications;

(vi) Discover common denominator for all remaining items;

(vii) Carry out common denominator conversions gathering all addition items together at the same time on the left-hand side and all subtraction items on the right-hand side;

(viii) Add together all addition items;

(ix) *Add* together all subtraction items;

(x) Subtract (ix) from (viii);

(xi) Cancel final statement as far as possible;

(xii) Reconvert from improper fraction to whole units and indicate as answer.

(*c*) It will be seen that apart from the relating of fractional parts to real life at the end of Class 3, the main work in fractions to date has been confined strictly to number. If the principles outlined above have all been clearly understood, however, no child should have any difficulty in going on to deal with problems involving fractions in other media such as money, length, weight and capacity.

DECIMALS

Aim

Introduction of decimals with addition and subtraction.

Step 1. Meaning of the first decimal place.

Begin by obtaining from the class a definition of a fraction and about six examples of fractions. Then say to them: 'For many people at their work—engineers who make motor cars or aeroplanes, the government treasurer and his assistants who work out the amount of taxes everyone should pay, the electrician who makes machinery to supply light to the big towns, the man who makes a radio set—the most important fractions are those whose denominator is 10, or 100, or 1,000, or 10,000. Look at one inch on your ruler. If you look at this edge of the ruler' (show the class) 'you will see that it has been divided off into tenths. Now try to imagine that you must divide each tenth into ten parts—it would be very difficult. But many people have to divide each tenth not only into ten but also into a hundred parts. They do it every day. So these fractions are very important for many workers. They are so important that there is a special way of writing them down, and a special name for them when they are written down in that way. We call them *decimals*.' (Now write this name on the blackboard.) 'When I want to show that I am writing a decimal, I simply write the number and put a full stop in front of it—like this . . . ·1, ·2. Notice the full stop is written half way up the number: and the full stop is called the *decimal point*.' (Write this on blackboard.) 'The numbers I have written I must read as "point one" and "point two". They mean $\frac{1}{10}$ and $\frac{2}{10}$.'

Your blackboard should now look like this:

DECIMALS

$\cdot 1 = \frac{1}{10}$ Decimal POINT

$\cdot 2 = \frac{2}{10}$

Now, by questioning the class, make up the blackboard to show the decimals ·1 to ·9, with the corresponding fractions ($\frac{1}{10}$ to $\frac{9}{10}$) written against them. When all are written on the blackboard, question round the class in this way:

(*a*) What is $\frac{1}{10}$ (etc. to $\frac{9}{10}$) as a decimal?

(*b*) What is ·1 (etc. to ·9) as a fraction?

(*c*) Mix questions of types (*a*) and (*b*).

This must first be done with the blackboard summary visible to the class. You may have to explain the questions you ask, but you *must* establish the form of question shown here, because it is the standard form of expression used in arithmetic. When the class has had a good drill with the help of the blackboard summary, rub it off and give the drill from memory.

(*d*) Questions as above with answers written in books.

Next write on the blackboard a mixed number, e.g. $2\frac{1}{10}$ (two and one-tenth). Ask the class: 'What is the figure 2?' They should tell you that it is two whole numbers or two units. Write it down *again*—2. Then ask: 'What is $\frac{1}{10}$ as a decimal?' The class should say ·1. Write it against the 2 so that your blackboard now reads:

$$2\tfrac{1}{10} = 2{\cdot}1$$

Repeat the same process with several mixed numbers each having tenths as the fraction, and build a blackboard summary like this:

$$3\tfrac{2}{10} = 3{\cdot}2 \qquad 5\tfrac{9}{10} = 5{\cdot}9$$

$$4\tfrac{7}{10} = 4{\cdot}7 \qquad 6\tfrac{3}{10} = 6{\cdot}3$$

and similar numbers, but be careful to have only units and tenths. Then question the class by the four-point method—(*a*), (*b*), (*c*), (*d*)—shown above.

When the four types of question have been dealt with, put back on the blackboard three or four of the mixed numbers, both with tenths and decimals as shown above, e.g.

$$3\tfrac{2}{10} = 3{\cdot}2 \qquad 5\tfrac{9}{10} = 5{\cdot}9$$

Now say: 'Look at three point two. How many figures are there after the decimal point (that means on the *right* of the

decimal point)? Yes, there is one figure. What does this figure mean? It means two-tenths.' (Repeat this form of questioning with several examples, so that the class become familiar with the expression 'after the decimal point', and with the fact that the first figure after the point is 'tenths'.) 'The first figure after the decimal point shows "how many tenths". It is the tenths place. It is called the *first decimal place*.' (Write on blackboard.) 'What is the first decimal place? . . . It is the tenths place. It is the first figure after the decimal point.' Now make sure that the class can remember this name and definition.

Step 2. Meaning of second decimal place.

Write on the blackboard the number 11·1. Point to the *unit* figure and establish that it means 'one unit or one whole number'. Having done so, point to the *tens* figure and establish that it means 'one ten'. Lastly, revise that the figure in the first decimal place means one-tenth. Then write this on the blackboard:

$$11{\cdot}1 = 1 \text{ ten} + 1 \text{ unit} + 1 \text{ tenth}$$

Next write the figures 222·2, and by the same kind of question build up the blackboard this way:

$$222{\cdot}2 = \text{two } \textit{hundreds} + \text{two } \textit{tens} + \text{two} + \text{two } \textit{tenths}$$

Now write the same figures as before but add a second decimal place:

$$222{\cdot}22$$

By reference to your previous blackboard summary, try to obtain, by questioning, the following statement of the new number:

$$222{\cdot}22 = 2 \text{ hundreds} + 2 \text{ tens} + 2 + 2 \text{ tenths} + 2 \text{ hundredths.}$$

Note: You must go through the stages shown already in this step.

Repeat this process with several numbers of the same type, e.g. 333·33 ; 666·66 ; 888·88.

Then repeat it with numbers in which the digits are not all the same, e.g.

246·86 ; 492·16 ; 127·43

With each example point to the tenths and recall the end of Step 1—the first figure after the decimal point is the tenths place ;

it is the first decimal place. And add 'The second figure after the decimal point is the hundredths place; it is the *second decimal place*'. After you have said this yourself once or twice, make the pupil who is helping with the statement of the number say it himself. In this way the class will learn the meaning of the expression.

Now make the class write in figures many expressions of this type:

One hundred and twenty-seven, three tenths and four hundredths; and in words *about five* numbers like 143·27.

Step 3. Relationship between first and second decimal places.

Begin by writing on the blackboard about five numbers like those used in Step 2. Ask the class to say what each means, but do not write the words down on the blackboard. Then rub out *all the figures in front of the decimal point* in each case, leaving simply the decimals written on the blackboard. Then ask for the meaning of each decimal and write that meaning on the blackboard in the vulgar fraction form. Your blackboard then might look like this:

$$\cdot 73 = \tfrac{7}{10} + \tfrac{3}{100}$$
$$\cdot 58 = \tfrac{5}{10} + \tfrac{8}{100}$$
$$\cdot 62 = \tfrac{6}{10} + \tfrac{2}{100} \quad \text{etc.}$$

Now ask the class to work out on the blackboard the 'addition of fractions' which already appears there, following the rules given in the chapter on Fractions in Class 3. They will then see that the expressions:

$$\tfrac{7}{10} + \tfrac{3}{100} = \tfrac{73}{100}$$
$$\tfrac{5}{10} + \tfrac{8}{100} = \tfrac{58}{100}$$
$$\tfrac{6}{10} + \tfrac{2}{100} = \tfrac{62}{100} \quad \text{etc.}$$

and therefore that

$$\tfrac{62}{100} = \tfrac{6}{10} + \tfrac{2}{100} = \cdot 62$$

and so on.

Once this has been grasped, and it should be easy, give a large number of examples, say 20, after the style of a mental arithmetic period, i.e. you read out the vulgar fraction and the class write it down as a decimal. Examples like this:

'Write as a decimal: $\tfrac{54}{100}$, $\tfrac{85}{100}$, $\tfrac{92}{100}$, $\tfrac{41}{100}$. . . .

Write as a fraction: ·87, ·95, ·19, ·75, ·62. . . .'

Step 4. Establishment of 0 in first decimal place.

Begin by taking a few examples from Step 3, asking the class to change vulgar fractions into decimals. Be sure in this that you only ask them to change fractions which have a 2-digit numerator, e.g. $\frac{27}{100}$. Then say:

'Now supposing I am asked to change $\frac{7}{100}$ into a decimal. I have no "tens figure" in my numerator. I could therefore write it $\frac{07}{100}$.' (Here revise the rule they have grown familiar with—that a 0 on the left makes no difference to the value of the number. Then you can show the different rule for decimals.) 'This shows me clearly that I have *no tenths* and seven hundredths. Now, when I write this fraction down as a decimal, I must show that there is nothing in the tenths column. I cannot leave it empty. I must put the 0 in to show that there are no tenths to go into the first decimal place. If I did leave that space empty, a person reading my sum might easily make a mistake and think that my figure 7 should have gone into the first decimal place and not the second. So I write it like this:

·07

That means "No tenths and seven hundredths" or simply "seven hundredths". Remember this rule: In decimals, *never* leave a blank space; *always put in the 0 if there is a figure in a place to the right.*'

Now follow the method of Step 1 to drill the second decimal place with 0 in the first decimal place—the class change $\frac{1}{100}$ to $\frac{9}{100}$ into decimals, and vice versa. Then give them mixed numbers to write down, a unit and some hundredths, e.g.

$9\frac{1}{100}$; $6\frac{4}{100}$ etc.

Step 5. Final 0 in decimals (0's on the right).

The next point to be established is that any number of 0's may be written after the last decimal place without altering the value of the decimal.

Revise first the fact learned in Step 4.

Write on the blackboard	·7
Ask the class what it means	7 tenths
Then add on the right a 0	·70
Ask the class for the meaning	7 tenths and 0 hundredths

Ask if the decimal is any bigger or smaller. (Get the answer that there is no difference.)

Repeat with all numbers from ·1 to ·9.

Next add two or three 0's to all your examples and confirm the class knowledge just learned.

Repeat the steps with 2 places of decimals.

Make as many examples of 2-place decimals followed by two 0's as you can.

Use questions all the time. Repeat after each example (changing only the numbers), the words underlined :

'·7 is 7 tenths. ·70 is 7 tenths and 0 hundredths. It is still only 7 tenths.'

or

'·68 is 6 tenths and 8 hundredths. ·680 is 6 tenths, 8 hundredths, and 0. It is still only 6 tenths and 8 hundredths.'

When this knowledge is firm, combine Step 4 and Step 5 so that the class learn the difference between—e.g.—·07 and ·70. Use all numbers from ·01 to ·90.

Now give the rule : 'Any number of 0's may be written after the last decimal figure—that is, on the right of it—without making any difference to the value of the decimal.'

ADDITION

Step 1. Demonstration and proof of method.

(*a*) With 1 decimal place.

Put the following sum on the blackboard.

$$\begin{array}{r} \cdot 7 \\ + \ \cdot 8 \\ \hline 1\cdot 5 \\ \hline \end{array}$$

Say : 'When adding decimals together we simply add the numbers as we did for simple addition long ago in Class 2. But we have now to watch that we are most careful to put all the decimal points exactly underneath one another.

'So before I add at all, I write the decimal point for the answer exactly underneath the other decimal points. Then I say

8 + 7 are 15.

Write the 5, carry the 1.

So I carry the 1 over to the left of the point.

There is nothing to add to it.

So I write the 1 in the units place of the answer.

My answer is 1·5.

'Let us prove that it is right—·7 is what? ($\frac{7}{10}$) and ·8 is ($\frac{8}{10}$) —right

$$\frac{7}{10}+\frac{8}{10}=\frac{15}{10}=1\tfrac{5}{10}=1{\cdot}5\text{ '}$$

Make one or two more examples for demonstration, e.g.

$$\cdot 9+\cdot 7;\quad \cdot 4+\cdot 8;\quad \cdot 6+\cdot 6$$

Be sure they all carry over the decimal point.

Now give the class not less than 10 examples to do themselves, and afterwards make them prove their answers by mental addition of the equivalent vulgar fractions.

(*b*) With 2 decimal places.

Now extend your demonstration and proof to addition of the second decimal place with a 0 in the first decimal place. Use exactly the same method as before.

Example:

$$\begin{array}{r} \cdot 07 \\ +\cdot 08 \\ \hline \cdot 15 \\ \hline \end{array}$$

Proof: $\frac{7}{100}+\frac{8}{100}=\frac{15}{100}=\cdot 15$

Give the class the same amount of practice in the same way as before.

Note: Do *not* give figures which add to 10, e.g.

$\cdot 7+\cdot 3$; $\cdot 04+\cdot 06$ etc.

Step 2. Addition of 2-place decimals carrying through, with units in answer.

Very little demonstration should be needed at this step, and

proofs can be dispensed with. Begin with examples of two items, then increase to three, lastly four.

$$\begin{array}{r} \cdot 96 \\ +\cdot 85 \\ \hline 1\cdot 81 \\ \hline \end{array}$$

Stress the rules for setting down, and give at least 20 examples in all for the class to do. Make some examples in which the answer to the first place addition (i.e. the answer in the tenths column) is 0. There is no need to demonstrate these specially, but be ready to help the backward pupils if they meet difficulty here.

Step 3. Addition of mixed numbers with 2-place decimals.

In this step do not have more than 3-digit whole numbers (i.e. up to hundreds) in your examples, e.g.

$$\begin{array}{r} 6\cdot 23 \\ +2\cdot 35 \\ \hline \end{array} \qquad \begin{array}{r} 16\cdot 42 \\ +13\cdot 39 \\ \hline \end{array} \qquad \begin{array}{r} 241\cdot 38 \\ +212\cdot 48 \\ \hline \end{array} \qquad \begin{array}{r} 251\cdot 64 \\ +132\cdot 36\,* \\ \hline \end{array}$$

* Include a few examples like this where the decimals add to ·00 among the 20 or more examples which you must now make for the class.

Step 4. Addition of 1-place to 2-place decimals.

These sums apply the rule learnt in Introduction, Step 5. Revise it first by questioning the class. Demonstrate these examples :

$$(a)\ \begin{array}{r} 39\cdot 4 \\ +48\cdot 36 \\ \hline 87\cdot 76 \\ \hline \end{array} \qquad (b)\ \begin{array}{r} 265\cdot 37 \\ +128\cdot 9 \\ \hline 394\cdot 27 \\ \hline \end{array} \qquad (c)\ \begin{array}{r} 123\cdot 07 \\ +\ 24\cdot 3 \\ \hline 147\cdot 37 \\ \hline \end{array}$$

Say (*a*) 6 and 0 make 6 . . . etc.

(*b*) 0 and 7 make 7 . . . etc.

(*c*) 0 and 7 make 7 . . . 3 and 0 make 3.

Now make at least 20 examples for the class to do, mixing types (*a*), (*b*) and (*c*).

SUBTRACTION

Step 1. Subtraction of 2-place decimal from 2-place decimal.

This step is simple and should be short. Its aim is to teach care in placing the decimal point.

Teach the class that if the decimal point is carefully placed in the answer line underneath the other points, the subtraction itself is the same as they did in Class 2.

$$\begin{array}{r} \cdot 92 \\ -\cdot 64 \\ \hline \cdot 28 \\ \hline \end{array}$$

Make at least 10 examples like that for the class to do.

Step 2 Subtraction of mixed number from mixed number with 2-place decimals.

(*a*) Whole number in top line only.

(*b*) Whole number in both lines.

(*a*) The point to be established is that a carried figure of 1 unit brought to the right of the decimal point becomes 10 tenths.

Put on the blackboard :

$$\begin{array}{rcc} & 2 & 5 \\ - & & 9 \\ \hline & & \\ \hline \end{array}$$

(Notice the spacing. *No* decimal point.)

Have a pupil do this sum as though he were showing Class 2 how to do it. When he says ' Take a 10 from my 2 tens ', revise with the class that by doing this, 1 ten becomes 10 ones (or units). Allow him to complete the sum. Then repeat the same sum, but first, put the decimal point in the space between the 2 and 5 ; and in front of the 9.

At the same point in the working out, show that one *unit* taken across to the right of the decimal point

becomes ten *tenths*. When this point is clear to all, give them practice.

$$\begin{array}{r} 2{\cdot}78 \\ -\ {\cdot}84 \\ \hline \end{array} \qquad \begin{array}{r} 2{\cdot}35 \\ -\ {\cdot}47 \\ \hline \end{array}$$

(*b*) No further explanation should be needed before going on to the main work of this step. When you make your own examples *do not* have any 0's in the decimals of the top line.

$$\begin{array}{r} 364{\cdot}57 \\ -178{\cdot}28 \\ \hline \end{array} \qquad \begin{array}{r} 675{\cdot}29 \\ -298{\cdot}58 \\ \hline \end{array} \qquad \begin{array}{r} 427{\cdot}52 \\ -\ 99{\cdot}06 \\ \hline \end{array}$$

Give 20 examples.

Step 3. Subtraction of mixed numbers with 0 in first decimal place of top line.

The explanation of the subtraction can be found in Subtraction of Number, Class 3, Steps 4 and 5, page 21 and 22. Repeat again the rule of placing the decimal point in the answer before beginning to subtract.

(*a*) 0 in top line only.

$$\begin{array}{r} 248{\cdot}07 \\ -\ 79{\cdot}43 \\ \hline \end{array}$$

and not less than 10 more examples.

(*b*) 0 in each line.

$$\begin{array}{r} 753{\cdot}02 \\ -387{\cdot}04 \\ \hline \end{array}$$

and not less than 10 more examples.

Step 4. Subtraction with 0 in second decimal place. Refer back again to Introduction, Step 5, page 22.

(*a*) One decimal place from two decimal places.

There is no real difficulty here. Demonstrate:

$$\begin{array}{r} 98{\cdot}45 \\ -36{\cdot}7 \\ \hline \end{array}$$

'5, take away nothing, leaves 5 . . . etc.'

(*b*) Two decimal places from one decimal place.

After referring again to Introduction. Step 5, establish the method by demonstrating:

$$\begin{array}{r} 63{\cdot}2 \\ -\,19{\cdot}36 \\ \hline \end{array}$$

Say: 'Six from nothing I cannot take. Write in the 0 and carry 1 from my 2 tenths. Now I can say 6 hundredths from 10 hundredths leaves 4 hundredths.'

Make sure that this method and form of words is well known. Stress again that the subtraction is the same as for ordinary numbers. Then give not less than twenty sums to the class for practice.

MULTIPLICATION

Step 1. Demonstration and proof of rule.

It is best to approach multiplication of decimals by first multiplying two vulgar fractions whose denominator is 10. So write on blackboard $\frac{3}{10} \times \frac{7}{10}$, and say:

'I want to multiply three tenths by seven tenths. Do this sum —So-and-so.' One of the pupils then does the working of the sum, following the rules for multiplication of fractions. Your blackboard will then look like this:

$$\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$$

Now say: 'But $\frac{3}{10}$ can be written ·3, and $\frac{7}{10}$ can be written ·7. If I multiplied ·3 by ·7 I should get the answer $\frac{21}{100}$ written as a decimal. What is seven times three? 21. But what is $\frac{21}{100}$ as a decimal? ·21. Now, how many decimal places are there in ·3 and in ·7? Yes, one in each. But how many are there in ·21? Yes, two decimal places. So I have one decimal place in the multiplicand (the top line), one in the multiplier (the bottom line), and two in the answer. One and one make two. So if I multiply the numbers as though there were no decimal point, that will give me the right *figures* for the answer: and then I write my decimal point in the answer so that there is the same number of decimal places in the answer as in the other two lines put together; that

will give me the right number of decimal places. Let us try another sum and prove that we are right:

'I want to multiply	$\cdot 9 \times \cdot 4$	So I write that as my first line.
Then I put down the figures	9 $\times 4$	
I multiply and I find that	36	is my answer.
I see that I have one decimal place in my multiplicand $\cdot 9$: and one in the multiplier $\cdot 4$:	$\cdot 36$	One and one make two. So I have two decimal places in my answer.

To prove my sum:

$$\tfrac{9}{10} \times \tfrac{4}{10} = \tfrac{36}{100} = \cdot 36$$

Now state the rule for working in the class books:

(*a*) Write down the sum as it is given.

(*b*) Do the multiplication without any decimal points.

(*c*) Count the number of decimal places in the multiplicand and multiplier altogether.

(*d*) Write in the decimal point so that there is the same number of places in the answer.

Step 2. Practice. Multiplication of 1-place by 1-place.

Give the class at least 20 examples like these to do by themselves, and make them prove their answers by multiplication of fractions:

$$\cdot 8 \times \cdot 7; \quad \cdot 9 \times \cdot 3; \quad \cdot 6 \times \cdot 4; \quad \cdot 9 \times \cdot 7, \text{ etc.}$$

Step 3. Multiplication of mixed number by decimal, 1-place in both lines.

The method has been established in Step 1, but this type of sum needs demonstration, and proof by the fraction method—of one or two examples—helps to make the matter clear. Say: 'If we wish to multiply mixed numbers and decimals, we use exactly the same method. What are the four steps of the method

we use?' (Put the answers on the board as a revision.) Then demonstrate:

$$\begin{array}{r} 8{\cdot}9 \times {\cdot}7 \\ 89 \\ \times 7 \\ \hline 623 \\ \hline \end{array} \qquad 6{\cdot}23 \ \textit{Ans.}$$

Proof: $8\frac{9}{10} \times \frac{7}{10}$

$$=\frac{89}{10} \times \frac{7}{10} = \frac{623}{100} = 6\frac{23}{100} = 6{\cdot}23$$

Give the class at least 20 examples to do by themselves. Ten of these should have multiplicands of less than ten: ten should have multiplicands of between 10 and 99. If these are well done, give a further ten examples where the multiplicand has four figures, including one decimal place, e.g. $156{\cdot}8 \times {\cdot}6$.

Step 4. Multiplication of mixed number by mixed number, 1-place in each.

Here you simply revise the four-stage rule for multiplication of decimals and the rules for long multiplication of number, and give the class ten examples of *each* of the following grades of sum:

(*a*) Two figures each line $7{\cdot}9 \times 6{\cdot}4$

(*b*) Three figures by two $24{\cdot}7 \times 3{\cdot}8$

(*c*) Three figures by three $35{\cdot}2 \times 26{\cdot}9$

(*d*) Four figures by three $248{\cdot}7 \times 25{\cdot}6$

Note: You must not have more than one decimal place in multiplier and one in multiplicand. Make sure the answer *never* ends in 0.

Step 5. Multiplication of whole number by decimal.

(*a*) One place of decimals in multiplier.

Again simply a matter of stressing the four steps of the method but with care to see that the class understand that since there are no decimal places in the multiplicand, the last stage of the sum is: 'One place and 0 place is 1 place, so I have 1 decimal place in the answer.'

$$5 \times {\cdot}9\,; \quad 37 \times {\cdot}7\,; \quad 246 \times {\cdot}6$$

Give the class five of each type of sum to do.

(*b*) Two places of decimals in multiplier.

Revise the four-stage rule and show that 2 places plus 0 places = 2 places of decimals in the answer.

$$9 \times \cdot 24 ; \quad 48 \times \cdot 39 ; \quad 197 \times \cdot 76$$

Again give five of each type to the class.

Step 6. Multiplication of decimal or mixed number by whole number.

By this time all demonstration should be done with class co-operation, as the principle should be clearly understood, and the teacher is simply showing variations of the same basic sum :

(*a*) Decimal by whole number : $\cdot 8 \times 7$; $\cdot 9 \times 23$; $\cdot 67 \times 35$.

(*b*) Mixed number by whole number : $9\cdot 3 \times 9$; $8\cdot 5 \times 45$ $9\cdot 76 \times 12$; $85\cdot 23 \times 19$; $312\cdot 43 \times 27$, etc.

Only the less bright children should require much help in detail at this stage. The class should be given ten examples of (*a*) and about twenty of (*b*) to do alone. Make sure that the answers never end in 0.

Step 7. Multiplication where product ends in 0 or 00.

The point requiring teaching here is that although 0's on the right of a decimal have no value, *they must be counted* when reckoning the number of decimal places in the answer to a multiplication sum.

Demonstrate the following multiplication of vulgar fractions (with class co-operation) :

$$\tfrac{7}{10} \times 10 = \tfrac{70}{10} = 7$$

Next do the same sum by the decimal method. The class will find the uncorrected answer :

$$\cdot 7 \times 10 \ldots\ldots\ldots\ldots 7 \times 10 = 70$$

Correct it yourself—one place of decimals in the original two numbers—the answer............$7\cdot 0 = 7$.

Repeat the demonstration with class co-operation, using $\cdot 1$ to $\cdot 9$ and multipliers 10, 20 and 30.

Next show the result of multiplying ·07 by 10 :

$$\frac{7}{100} \times 10 = \frac{70}{100} = \frac{7}{10}$$

and ·07 × 10..................7 × 10 = 70..................

2 decimal places..................

·7

Repeat the demonstration as before.

Now multiply one-place and two-place decimals by 100 in the same way.

Take now sums wherein two other figures multiply together to give an answer ending in 0 :

(*a*) ·4 × 5, ·5 × 4, ·8 × 2..................1 decimal place.

(*b*) ·4 × ·5, ·05 × 4, ·8 × ·2, 6 × ·05..................2 decimal places.

Leave a good selection of these sums on the board.

Draw the attention of the class to the answers and make the rule :

> ' 0s at the end of the answer to a multiplication sum *must* be counted when reckoning the number of decimal places in the answer.'

Refer again to Introduction, Step 5.

Point out the sums on your blackboard which have 0s but no *figures* after the decimal point.

Make the final form of the rule for 0s in the decimal places :

> ' 0s in the decimal places have no value and make no difference to the decimal unless there is a figure to the right of the 0s.'

Step 8. Examples.

Now demonstrate each of the following sums :

(*a*) 38·8 × ·5

(*b*) 7·6 × 5·5

(*c*) 352·4 × 24·5

(*d*) 230 × ·76

(*e*) 9·62 × 200

Finally, make five examples of each of these types for the class to do themselves.

Note: Be very careful to see that this class is never given a sum which requires them to *insert* a 0 after the decimal point when correcting the answer, e.g. ·2 × ·4.

DIVISION

Step 1. Division of mixed numbers by a whole number.

(*a*) Short division.

Teach the rule: Put the decimal point in the answer line underneath the decimal point in the dividend before starting to divide. Then divide as in ordinary number sums.

Demonstrate an example with one, and one with two, decimal places.

```
(i)  12)98·4        (ii)  9)628·38
        ----                -------
         8·2                 69·82
        ----                -------
```

(*b*) Long division.

Rule: As for short division but *over* instead of underneath.

Demonstrate one example with 1 decimal place and one with 2.

```
       328·9              43·18
    --------           --------
 58)19076·2         67)2893·06
    174                268
    ---                ---
     167                213
     116                201
     ---                ---
      516                120 †
      464                 67
      ---                 --
       522 *              536
       522                536
       ---                ---
       ...                ...
```

* Say: '52 remaining. Bring down the 2 tenths from the first decimal place.'

† Say: '12 remaining. Bring down the 0 from the first decimal place.................53 remaining. Bring down the 6 hundredths from the second decimal place.'

Make 20 examples for short division and 20 for long division for class practice.

Here is the easiest way to make them :

Take two numbers (one under 13 for short division).

Multiply them together.

Write down the answer with a decimal point before the last figure or the last two figures.

Let the class divide this number by the lower of the two original numbers, e.g. $243 \times 12 = 2916$. Write this as 29·16.

The class do the sum.................. $29{\cdot}16 \div 12 = 2{\cdot}43$.

Note: Be sure the product of your own multiplication never ends in 0 or 00.

Step 2. Division of number, with or without a decimal point, by a number with a decimal point. Answer : a whole number.

The next step to be taught is that when decimals are divided by decimals or by mixed numbers, the rule is : 'Change the divisor into a whole number and move the decimal point in the dividend the same number of places in the same direction.' Before this rule can mean anything, the class must know what is the effect of moving the decimal point and must see that equally multiplied numbers, divided one into the other, produce the same quotient as the numbers themselves. Proceed in this way.

(*a*) Making the divisor a whole number by multiplication.

Ask the class to do the following sums in their books :

$$4\overline{)832} \qquad 40\overline{)8320} \qquad 400\overline{)83200}$$

and then ask what they notice about the answers. They will say that all the answers are the same. Then ask the class what they notice about the three divisors, and the three dividends, and build a blackboard summary as follows :

$$\begin{array}{ll} 832 & \div 4 \\ 832 \times 10 & \div 4 \times 10 \\ 832 \times 100 & \div 4 \times 100 \end{array}$$

Then make the rule with the class :

'If in a division sum we multiply divisor and dividend by the same number and then divide, we get the same answer as that produced by our original numbers.'

You cannot spend too much time on making this point clear if you want to save endless trouble later on. Make the class do a large number of examples, using common multipliers of 10 at first, and then, for instance:

$$49 \div 7$$

$$49 \times 3 \div 7 \times 3, \text{ etc.}$$

Now go on to show that the same principle applies to decimals. Like this:

'Take as your division	$8{\cdot}4 \div 2{\cdot}1$
Multiply each side by 10	$84 \div 21$
Work out on blackboard by long division	4 *Ans.*

To prove. Multiply the divisor 2·1 by the answer 4.
There is one place of decimals in 2·1.
There is no place of decimals in 4.
So the answer is 8·4

and the method of division is correct.'

(*b*) Making the divisor a whole number by moving the decimal point.

Take one of the sums used as an example in section (*a*), for example:

$$\cdot 07 \times 10 = \cdot 7$$

Write it on the blackboard in the form given here, and below it write

$$\cdot 07 \times 10 = 0{\cdot}7$$

Repeat with 100 so that the blackboard reads:

$\cdot 07 \times 10 = \ \cdot 7$	$\cdot 07 \times 100 = 7\cdot$
$\cdot 07 \times 10 = 0{\cdot}7$	$\cdot 07 \times 100 = 07\cdot$

The class will see now what you mean when you give the rule:

'To multiply a decimal by 10, move the decimal point one place to the right. To multiply by 100, move the decimal point two places to the right.'

Now revise the rule made in the previous section (*a*), and ask the class whether it is enough to multiply only one side of the

division sum. They will answer that both sides must be multiplied by the same figure. So, in the sum

$$96 \div 1{\cdot}2$$

if I multiply the divisor by 10, doing so by moving the decimal point one place to the right, I must also multiply the dividend by 10. Ask what is the new dividend. The class will say 960.

Put on the blackboard 96 and 96·0.

Ask if these are the same—the class will say 'Yes'.

Show now that if we write our sum

$$96{\cdot}0 \div 1{\cdot}2$$

we can multiply both numbers by 10 by moving the decimal point one place to the right. Make this point clear by doing several examples of correction in the way just described. Then state and ensure that the class learn the rule:

'To divide a number by a number containing a decimal point, make the *divisor* a whole number by moving the decimal point to the *right*; and move the decimal point in the *dividend* the *same number* of places in the *same* direction.'

(*c*) Practice of the step.

Demonstrate each of the sums shown below and give five examples of each type for practice.

Short division when corrected divisor is under 13.

Daily revision of the rule is needed.

Give practice after each *pair* of examples has been demonstrated.

(i) $280{\cdot}8 \div 3{\cdot}9$	$356{\cdot}4 \div {\cdot}9$
(ii) $84 \div {\cdot}07$	$2592 \div {\cdot}27$
(iii) $18{\cdot}75 \div {\cdot}25$	$3389{\cdot}1 \div 1{\cdot}43$

Step 3. Division with decimal place in the answer.

In this step the corrected dividend has a decimal place.

Give the rule for setting down:

'Copy the sum into your books as it is given. Correct the divisor and the dividend mentally. Write down the sum as you will divide it. Put the decimal point in the answer line

directly over (or under in short division) the decimal point in the corrected dividend. Divide as shown in Step 1.'

Example :

Given sum.......................159·39 ÷ 9·9

Sum to be worked :

```
        16·9
   99)1593·9
       99
       ---
       603
       594
       ---
        99
        99
        --
        ..
```

Note : The children copy the *figures*, not the words.

Give not less than 10 examples for the class to do by themselves.

Step 4. 'Bringing down a 0.'

This is the type of sum where the class have to put into practice the rule that '0s on the right of a decimal make no difference to the value'. Revise the rule with them before you begin. There are two main types of sum—first, where the 0 comes from the second place of decimals, and secondly where it comes from the first—i.e., where there is no decimal at all. When these are understood separately, sums may be given where there is no decimal in the dividend and the answer goes to two decimal places.

(*a*) Division of dividend with one decimal place after correction, two decimal places in answer.

38·55 ÷ 7·5

```
      5·14
   75)385·50      Revise rule for correction.
      375
      ---
       105
        75
       ---
       300 ..................Say '75 into 30 I cannot.
       300                   But what is the 30? It is
       ---                   30 tenths because the last
       ...                   figure I brought down was
```

the 5 from the first decimal place. Now, if I put a 0 against the 5 in the first decimal place (put it) does it make any difference to my decimal? No, it is still the same. Now, suppose I bring that 0 down and write it against my thirty tenths, what have I? Three hundred . . . what? 300 *hundredths*, because the second decimal place is hundredths. And thirty tenths is 300 hundredths. 75 into 300 goes 4 times.'

Now prove your answer by multiplying 5·14 by 7·5, and give the rule: 'When you have a remainder that you cannot divide in a *decimal sum*, add a 0 to it, divide, and put the answer in the next place of decimals. You can do this as long as there is anything left to divide.'

You should give the class at least ten of these sums to do now. The original dividend must have one more place of decimals than the original divisor.

To make examples, take a number (which will later become the answer of the pupils' sum) ending in 2, 4, 6 or 8 and another number (which will become the divisor of the pupils' sum) ending in 5. Multiply them together. Cross the final 0 from the answer. Put a decimal point in this answer and in the number chosen to be the divisor.

Note: If the divisor is a whole number, the dividend needs 1 decimal place. If the divisor has 1 decimal place, the dividend needs 2 decimal places.

E.g. $738 \times 325 = 239850$ *

Cross off the last 0: 23985

This gives four possible sums for the class:

$2398{\cdot}5 \div 738$ or 325

$239{\cdot}85 \div 73{\cdot}8$ or $32{\cdot}5$

* Watch that you do not get a double 0 at the end of this multiplication.

(*b*) Division of whole number by whole number, or of decimals both corrected to whole number, with one-place decimal in answer.

Begin by revising, from Step 2, Subtraction of Decimals, page 219, that a unit taken across a decimal point becomes ten tenths.

Then the final rule for 0s in decimal places in Multiplication of Decimals, Step 5, page 223.

Lastly, question the class to revise their knowledge of former ways of expressing a remainder. Take this sum :

$$45\overline{)171}$$

and work it out to show first the result, 3 r 36 : then the method by which this was shown as $3\frac{36}{45}$, and then take the same sum and tell the class that we are now going to work out our remainder as a decimal.

$$\begin{array}{r} 3{\cdot}8 \\ 45\overline{)171{\cdot}0} \\ 135\phantom{{\cdot}0} \\ \hline 36{\cdot}0 \\ 36{\cdot}0 \\ \hline \ldots \end{array}$$

360 .... Here refer to the previous step and to the rule quoted at the top of this section.

The other types of 'given numbers' are as follows :

16·9 ÷ 6·5 which corrects to 169 ÷ 65
236·6 ÷ 3·25 which corrects to 23660 ÷ 325
3·29 ÷ ·35 which corrects to 329 ÷ 35

You should make at least 10 examples of these sums for the class to do by themselves. The method of making them is the same as in section (*a*) of this Step 5.

Step 5 (*c*). Division as Step 5 (*b*) with 2-place decimal in answer.

This is a combination of Steps 5(*a*) and 5(*b*). If you make examples by the method given above, change the previous rule and be sure that there *are* two 0s at the end of your multiplication, and that neither of these appears in the corrected dividend, e.g.

75 × 60 = 4500, which gives as a possible sum
4·5 ÷ 7·5 or 45 ÷ 75.

375 × 24 = 9000, which gives as possible sums
·9 ÷ 3·75 or 9 ÷ 37·5 or 90 ÷ 375
·9 ÷ ·24 or 9 ÷ 2·4 or 90 ÷ 24

Demonstrate one of each type of example given and then give the class at least ten examples to do by themselves. Mix the

types of sum together. Work out your explanation from the models given in sections (*a*) and (*b*) of this Step 5. Insist on care in the placing of the decimal point in the answer line, paying special attention to the writing in of the point when the dividend has been corrected to a whole number.

When this step is properly understood, give at least one and preferably two periods of revision-practice on the division of decimals.

Finally, give a revision-practice on the whole of the topic.

UNITARY METHOD

The Unitary Method, as well as being a method in itself, is a preparation for the doing of Proportion at a later stage and helps very considerably in the understanding of Proportion.

It is advisable that the word Proportion is *not* used in Class 5 so that the children do not become confused as some are likely to do if the two terms are used at about the same time. Once the Unitary Method is well established, the children will more easily grasp that Proportion is merely a shortened written form of the same mechanical process.

Step 1. Early teaching of this method must be restricted to one type, preferably the type involving money, or cost.

Start the lesson with a good mental drill of the following nature in which the class is required to find the cost of one (a unit) article :

3 pens cost sh. 6, what is the cost of 1?

5 books cost sh. 15, what is the cost of 1?

10 debes of paraffin cost sh. 120, what is the cost of 1?

8 tins of paint cost sh. 56, what is the cost of 1?

and so on.

These will present no difficulty in a Class 5 and will provide a firm basis for the introduction of this new sum.

Step 2. Now go back to the first mental sum given and complete it, writing on the blackboard, e.g.

3 pens cost sh. 6, what is the cost of 5 pens?

Ask the class for the answer. Let all those who wish to give an answer do so. If some children succeed in giving the right answer, the class will be interested at once.

Now put the second sum of the mental drill on the blackboard in a complete form, e.g.

5 books cost sh. 15, what is the cost of 3?

Again ask for the answer.

It is time now to lead on the children, who have still not grasped the method, to understanding it. Say to the class.

'You will remember I gave you a sum like this before in which I asked if 5 books cost sh. 15, what is the cost of 1?

'What was the answer to that?' 'Sh. 3.'

'What, then, is the cost of 3 books?' 'Sh. 9.'

'You will notice that by finding the cost of one, from the facts you are given, you are then able to find the cost of any number. Let us write this sum out on the blackboard in the way we thought it out. The first thing we thought was "5 books cost sh. 15", so we write it on the blackboard. Then we find the cost of 1 book, so we write "1 book costs sh. 3", and from that we were able to give the cost of 3 books, so we write "3 books cost sh. 9".'

Thus the blackboard reads:

5 books cost sh. 15

1 book costs sh. 3

3 books cost sh. 9.

Now say to the class: 'Let us see how you worked it out and write it that way on the blackboard. How did you find the cost of 1 book?' When the answer is given that sh. 15 was divided by 5, write sh. 15 again on the right-hand side of the blackboard opposite the first line of the sum above, and ask a child to put sh. $15 \div 5$ *as a fraction* opposite the second line on the blackboard. Point out that you are writing the answer part of each line in another way. If fractions have been taught well, this will present no difficulty. The teacher must be sure that the children do understand that sh. $15 \div 5$ is sh. $\frac{15}{5}$. Now ask the children, 'How do we get the cost of 3 after finding the cost of 1?', and get a

child to give the answer part of the third line which is then written on the blackboard. The blackboard now reads :

5 books cost sh. 15		sh. 15
1 book costs sh. 3	or	sh. $\frac{15}{5}$
3 books cost sh. 9	or	sh. $\frac{\cancel{15}^{\,3}}{\cancel{5}} \times \frac{3}{1} =$ sh. 9

Explain to the children that it is better to use this fraction method, leaving the cancelling out until the end, because it will be simpler when numbers become more difficult, and instead of having two steps for working out an answer (one after the second and one after the third lines), we have one only (at the end of the third line).

Now question the class as follows :

Question : ' What did the sum ask? '
Answer : ' It asked for the cost.'
Question : ' Was the answer, then, shillings or books? '
Answer : ' Shillings.'
Question : ' On which side of each statement do you see shillings? '
Answer : ' On the right.'

Now give the **first rule :**

' *What is required by the answer is always written at the end of each statement.*'

Bring to the notice of the class now the **second rule :**

1. ' The answer to the first statement is always the first part of the answer to the second statement.
2. ' The answer to the second statement is always the first part of the answer to the last statement.'

So in our sum we had as our answer :

To the first statement	sh. 15
To the second statement	sh. $\frac{15}{5}$
To the third statement	sh. $\frac{15}{5} \times 3$

Step 3. On the blackboard do another sum from the earlier mental drill, e.g. 10 debes of paraffin cost sh. 120, what is the cost of 14 debes?

At this stage, the statements and answer parts should come from the children under the teacher's promptings and guidance, with the teacher writing the sum on the blackboard.

When this has been finished, again point out that parts 1 and 2 have been followed.

Give at least 20 sums of this kind, requiring *cost only*, until it is clear the class has thoroughly grasped it.

Step 4. Put the following sums on the blackboard:

6 books cost sh. 72. How many books will I buy for sh. 120?

Ask the class what is required in the answer. When told number of books emphasise that it is the *number* of books this time, not cost, that it is 'how many' instead of 'how much'.

Ask the class:

'How can I write the statement so that I obey Rule 1?' When this has been given, point out that in the sum's question form it is sometimes given in a different order from the way it must be written down, with reference to this sum. Then get the second and third statements from the class so that the blackboard reads:

sh.	72	will	buy	6 books
,,	1	,,	,,	$\frac{6}{72}$ books
,,	120	,,	,,	$\frac{6}{72} \times \frac{120}{1}$ books $= 10$ books

Give at least 10 sums like this in which number is required. Then give at least 20 mixed sums, 10 like Step 3 and 10 like Step 4.

Step 5. Sums involving weight, capacity and length (in place of money).

(*a*) Put the following sum on the blackboard (weight):

12 boxes of pencils weigh 2 lb., how much will 20 boxes weigh?

Ask the class to give the first statement, reminding them to note what is required in the answer. Write it on the blackboard.

12 boxes of pencils weigh 2 lb.

Go on in the same manner to the second and third statements and the completion of the sum on the blackboard.

Give 5 sums like this requiring weight.

(*b*) Put the following sum on the blackboard (capacity):

6 gal. 6 pt. of water are held in 3 tins; what will 5 tins hold?

Ask the class to give the first statement and write it on the blackboard.

3 tins hold 6 gal. 6 pt.

Now point out to the class that if we leave this statement in this form, we will have gal. and pt. in our final fraction and cancelling will be made difficult. Thus establish the rule that when two quantities are involved in the answer part of the first statement, they must be reduced to one quantity. In this case 6 gal. 6 pt. is reduced to 54 pt.

The sum on the blackboard is now altered to read:

3 tins hold 54 pt.

Go on to the second and third statements and the working of the sum so that the blackboard reads:

3 tins hold 54 pt.
1 tin holds $\frac{54}{3}$ pt.
5 tins hold $\frac{54}{3} \times \frac{5}{1}$ pt. $=90$ pt. $=11$ gal. 2 pt.

Point out that the final answer must be changed back to the two quantities; it may even be three quantities.

(*c*) Write the following sum on the blackboard (length):

10 tables together stretch 12 yd. 6 in. How far will 16 tables stretch?

Do this on the blackboard with the children in the same way as in (*b*) and then give 5 sums like it for practice.

Step 6. Fractional sums in which we are given the value of a fraction and have to find the value of the whole.

$\frac{5}{8}$ of a tank holds 20 gal.; how much does the whole tank hold?

Ask the class for the first statement, which is written on the blackboard. Go on to the second statement and third statement so that the blackboard reads:

$\frac{5}{8}$ of a tank holds 20 gal.
$\frac{1}{8}$,, ,, ,, $\frac{20}{5}$ gal.
$\frac{8}{8}$,, ,, ,, $\frac{20}{5} \times \frac{8}{1} = 32$ gal.

WORK SUMS

It will be noticed that the sums which could be called 'Work' sums have been omitted so far, e.g.

If 5 men dig a trench in 2 days, how long will 3 men take?

It may be said that up to this stage sums of Quantity and Measures have been dealt with—money, weight, capacity, length. It is advisable to do Work sums last because they involve an opposite process in the unit line or second statement (and similarly in the third statement). Whereas in the earlier sums we have always divided to find the answer, e.g.

When 5 books cost sh. 15, 1 book cost sh. $\frac{15}{5}$.

now it will be necessary to multiply.

Step 1. Discussion with the class.

Take the class outside. Show it a piece of work that needs to be done around the school, e.g. filling in holes, digging a plot in the garden.

Now say: 'Suppose I told you two boys (indicate 2 here) to do this job which should take you an hour. When I go back to the class-room one of you runs off and leaves the other to do the task alone. Will this boy finish the task in an hour?'

The class will readily answer no, that he will only have done a half. Thus the teacher proves in practice that it takes 1 person longer to do a task than it would take a group of persons.

Ask the class, now, how long it would have taken the one boy to do the whole task—2 hours.

Repeat this story with a different task and 3 boys, of whom 2 run away, and get from the class the answer that it would take 1 boy 3 times as long to do the task.

Establish the rule that it takes 1 person longer to do a task, so we *multiply* the time by the number of people who were first given the task.

Now give thorough mental drill to practise this rule, as follows:

3 men take 12 days to dig a hole, how long will 1 take?

A plot is hoed in 20 days by 4 men, how long will 1 take?

7 men took 14 days to pick some coffee, how long will 1 take? and so on.

Step 2. Establishing the second statement and the rule of multiplication in the written form.

After Step 1 has been done sufficiently well and it is clear that all children understand, write on the blackboard, *at one side* :

3 men dig a hole in 12 days

1 man digs „ „ 12×3 days

Emphasise here that we are doing the sum just as we have been doing the others, that we need 'days' in our answer, so we arrange the statement to have days on the right-hand side.

Now say to the class : 'Let us do a sum like those we have done before. If 5 books cost sh. 15, what is the cost of 1?' On the other side of the blackboard, opposite the Work sum, write the statement of this sum as a child dictates it. So the blackboard reads :

3 men dig a hole in 12 days	5 books cost sh. 15
1 man digs „ „ 12×3 days	1 book costs sh. $\frac{15}{5}$

Question : 'What difference do you note between the second lines of each sum?'

Answer : 'It was necessary to multiply in one and divide in the other.'

Question : 'Why did we multiply in the first sum?'

Answer : 'Because 1 man takes *more* time than 3 men.'

Question : 'How can you be sure whether to multiply or divide?'

Answer : 'We must first ask ourselves "Will the answer be more or less?" If more, we multiply ; if less, we divide.'

This cannot be stressed too much—stressed well, it will prevent many mistakes.

Step 3. Establishing the third statement and the rule of division.

Put this sum on the blackboard :

3 men take 12 days to dig a hole, how many days will 4 take?

Get children to do the first and second statement so that the blackboard reads :

3 men dig a hole in 12 days

1 man digs „ „ 12×3 days

Question : 'Will 4 men take more or less time than 1 man?'
Answer : 'Less.'
Question : 'Will I divide the answer part of line 2, or multiply?'
Answer : 'Divide.'

So the teacher completes the sum with the third statement as follows :

$$5 \text{ men dig a hole in } \frac{\overset{3}{\cancel{12}} \times 3}{\underset{1}{\cancel{4}}} \text{ days} = 9 \text{ days}$$

Follow this with one or two further blackboard examples, getting children to do this drill at the end of each line.

'*Is the answer more or less?*'

Insist that when the sums are done in the exercise books, this question must be asked by each child. When the teacher is sure the whole class has understood fully, he will give at least 20 Work sums for practice. When these are well understood, at least two revision periods will be given in which children can practise both Quantity and Work sums.

At this point the class could be told that this kind of sum is called the Unitary Method. This name is given because the second statement begins with 1 unit.

AREA

It will be necessary for the teacher to make arrangements for supplying the children with square inch pieces of cardboard. The teacher may find it necessary to require children to bring pieces of cardboard ; he can in a lesson get the children to draw the horizontals and verticals and make inch squares and then have them cut. Too much emphasis cannot be laid on the essential need for the practical and visual approach to the understanding of area. Blackboard demonstration with application may be enough for the brighter children but will not be sufficiently clear for most.

Bring to the attention of class the need to measure, not merely length, but also a surface—introduce little factual problems—if we wished to cover our class-room floor with biwempe carpet, what size would we want? Again, if we wished to know which is

the larger of two pieces of wood (here bring two pieces of wood, or cardboard, say 4 in. × 2 in. and 6 in. × 1 in.), how could we find out? Ask the class for its opinion on the examples brought before the class ; the class will probably be divided in its opinions. The teacher will bring out his square inch piece and before the class will place eight of them over the 4 in. × 2 in. piece. He asks : 'How many of these square inches are necessary to cover this surface?'—'Eight'. He will do the same with the 6 in. × 1 in. piece of wood and so the class is able to see that the 4 in. × 2 in. is a larger surface. 'How much larger?' 'By two square inch pieces.' The teacher goes on to explain that this is referred to as being larger by 2 square inches.

The children should now be required to find out how many square inch pieces will be required to cover as far as possible objects in the room, e.g. exercise book, text book, desk top, and so on, and write them down as follows :

Text book = 18 sq. in., etc.

The class should be told to remember that their answers are not absolutely accurate if the square inch pieces do not cover entirely the object.

After practical work of this type, the teacher now proceeds to the blackboard. His aim will be to get his children to see and understand that multiplying length by breadth will produce the size of the surface which is called *area*, but it must be done with care and the children gradually led to see for themselves. The teacher draws a rectangle 3 in. × 2 in. on the blackboard and marks off in inches vertically and horizontally. Bring to the notice of the class that the surface is made up of 2 rows of square inches, with 3 in each row, and its area is 6 sq. in., that is to say, the number of squares in a row multiplied by the number of rows ; in this case 3 in. × 2 in. = 6 sq. in. The teacher will carefully point out how in a way the L of 3 in. gives 3 squares, and the B of 2 in. gives 2 rows, and that by multiplying L by B we arrive at the answer without the need of having actually to divide the surface into squares, and count them. Set the class to draw rectangles, 8 × 5, 4 × 3, 6 × 2, with squares marked, and see for themselves. Underneath they will write 8 in. × 5 in. = 40 sq. in., etc. Now the class can find out the exact area of the objects previously

measured by square inch pieces by multiplying length by breadth to the nearest inch. Give mental work, e.g. what is the area of a desk top 16 in. × 8 in?, etc.

Now bring to the children the problem of having to find for the area of larger surfaces a more convenient unit than a square inch. Give the class the problem—' Let us find out the area of our class-room floor '. Get a child to measure the class-room, e.g. 25 ft. × 20 ft. The teacher points out : ' We have measured areas so far that are small and so it has been done in square inches. Now we have a much larger surface whose length and breadth is measured not in inches, but in feet. Do you think we can find a more convenient measure than square inches? ' At least some of the children will answer : ' Measure in square feet.' The teacher should then demonstrate (by a stick or chalk) by marking off a part of the floor in square feet—this would be sufficient to make the duller children understand the principle of the square foot.

This is a suitable stage to get from the class that the class-room and large surfaces can be measured by a square yard.

Obtain from the class the number of square inches in a square foot. If there is any failure in the class to give the answer, draw a square foot on the blackboard with its square inches. Now obtain from class how many square feet in a square yard by the principle of 3 × 3 (or 3 squares × 3 rows in B).

Give mental work on the three square measurements, of square inch, square foot and square yard.

E.g. What is the area of a room 12 ft. × 9 ft?

Which is the bigger room, one 12 ft. × 11 ft. or one 10 ft. × 13 ft.?

What is the area of a shamba 80 yd. by 20 yd.?

The class should now be ready for simple area sums.

TIME

Apparatus. At least one ordinary clock with a reasonably large dial (or failing this a model) and the double clock illustrated below on page 244. This latter should be made of wood, anything from 15 in. to 24 in. in diameter, and clearly painted as

shown. The two hands should be attached to the centre and be loose enough to be moved around, but not so loose that they fall out of position of their own accord when the clock is lifted up for demonstration purposes.

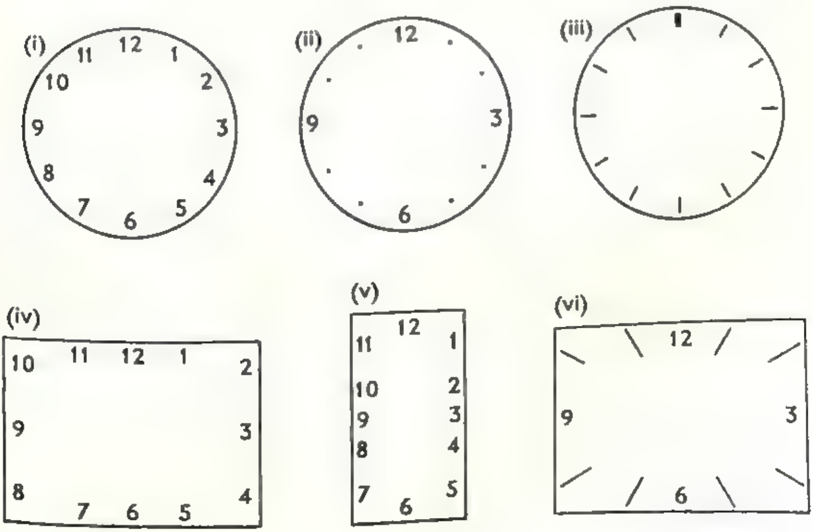
Step 1. The telling of clock time simply in hours (in English): 1 lesson only.

Note: This and some of the steps that follow below are really more of a language than an arithmetic lesson, and if the same teacher takes the class throughout the week, he should arrange for such steps to be taken in the English lesson *before* the arithmetic lesson in which he expects to start dealing with time.

In this case this first English-arithmetic lesson on time might proceed as follows:

(*a*) Introduction and explanation of vocabulary if not already known: hands, clocks, o'clock, hour hand, minute hand (do not worry too much about this one at present but mention it in passing), clock face (dial), etc.

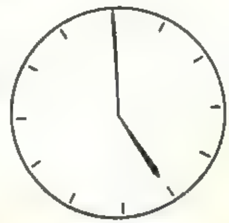
(*b*) Study of different types of clock and watch faces. The teacher should collect as many real specimens as possible of different types, but if some are not available they should be illustrated on the blackboard by drawings as follows:



The children should be made to realise from this that all clock faces have the same 'meaning' and position, with 12 always being 'in the middle at the top', and so on.

(*c*) Practise in English of the telling of the time in hours only.

(i) The teacher sets the clock at a particular hour, e.g.



and asks:

'What time is it now?'

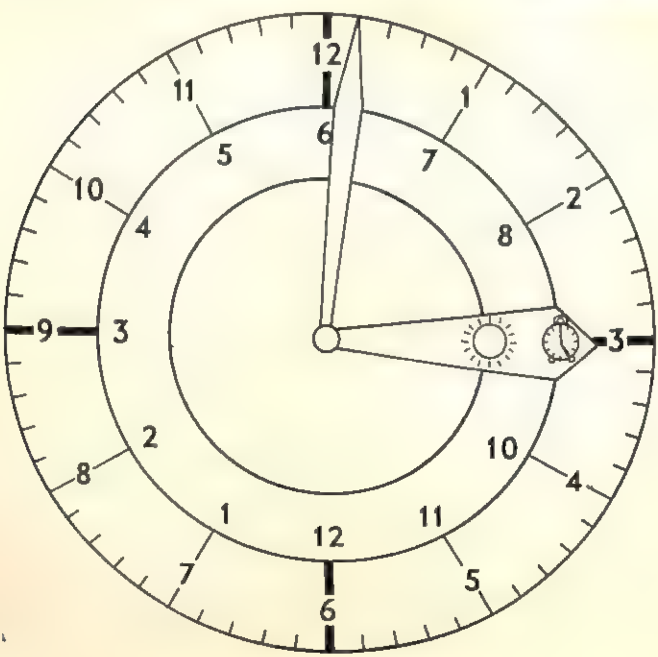
(5 o'clock.)

The teacher then winds the hands round to another hour (*note*: the mechanism of a real clock should be stopped) and repeats the process.

'What time is it now?' (11 o'clock.) Continue this until the children are familiar with all hours, positions *and* with the expression in English associated with each one.

Step 2. Comparison of clock time and sun time. (Again probably about 1 lesson only.)

For this it is absolutely essential that the double clock men-



tioned above under 'apparatus' be available. With this it is quite simple to make and learn the comparison between vernacular time and clock time.

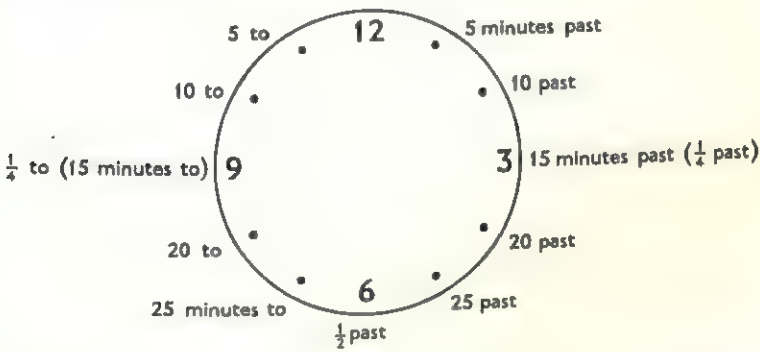
Points to note are:

(i) Deal with hours only at this stage.

and (ii) Remember sun time is *always* expressed in the vernacular and clock time *always* in English.

Step 3. Introducing fractions of hours, halves and quarters.

(*a*) Half hours. This can be done by reverting to one of the clocks used in Step 1 above and demonstrating the half hour. Explain that this always comes after or later than the hour to which we refer:



and is really 'one and a half hours' or 'half an hour after 1 o'clock'. Make it quite clear, however, that we do not use either of these terms, but always say 'half past one', 'half past nine', etc.

Note: Never 'half past nine o'clock': this is grammatically and historically correct (half past nine of the clock) but is never used nowadays in conversation in English.

(*b*) Quarters.

(i) Quarter *past*. This is the same as halves in (*a*) above and should be dealt with in the same way.

(ii) Quarter *to*. Explain that this comes *before* the hour to which it refers and demonstrate as for quarter past.

(iii) Ensure that the children all understand clearly the difference between *to* and *past* by using the clock and asking plenty of questions, mixing the quarters ' to ' and ' past ' all the time.

(*c*) Children to practise time recognition in hours, halves, and quarters.

Teacher draws clock faces on the blackboard as follows :

The children work out the time and write the answers in their books or for a mental test.

Four o'clock, quarter to five, quarter past eleven, etc.

Twenty examples to be done.

(*d*) If it is desired, a further comparison can now be made between sun time and clock time, including, on this occasion, fractions of hours. If this is done, the double clock as used in Step 2 above should be employed again.

Step 4. Introduction of minutes. (Again, all of this could well be done in an English lesson, if desired.)

(i) Apparatus required : the double clock and the usual classroom demonstration clock.

(ii) By turning the demonstration clock round an hour at a time, i.e. from 3 to 4, it should quickly be possible to elicit the connection that for every hour moved by the small (hour) hand, the big hand makes one complete circle.

(iii) Demonstrating now with the double clock (the ordinary ones are too small for all the children to see) it is possible for children to see (and count) quite clearly that there are 60 small dots to which the big hand points in turn, and these are spaced equally all around the edge of the clock. Starting from the top, the big hand moves past the first

(clockwise) and then right round to the last one at the top again (the 60th).

Now it is easy to explain that an hour is made up of 60 minutes and each of these small divisions represents a minute.

(iv) Reference to the five times table should soon reveal that the big (hour) numbers give us a quick way of adding up minutes as the distance between each of the hour figures is 5 minutes ($60 \div 12 = 5$).

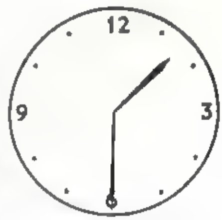
(v) Teacher to explain and establish the facts that—

(*a*) We speak of *past* the hour for the 30 minutes from the figures 12 to 6 (clockwise) and *to* the hour for the 30 minutes from 6 onwards round to 12.

(*b*) We count *upwards* in minutes *past* the hour, e.g. 5 minutes past, 10 minutes past, etc., *but* we count *downwards* in minutes *to* the hour, e.g. 25 minutes to, 20 to, etc.

(*c*) That 15 minutes to and 15 minutes past the hours are the quarters, and we more usually say ' quarter to ' rather than ' 15 minutes to ', but either may be used.

(*d*) Illustration of (*a*), (*b*) and (*c*) above on the blackboard as follows:



The teacher rubs off the outside explanations and tests to ensure that the children know this well.

(vi) Teacher to set and the children to do 30 examples as in Step 3(*c*) above, but this time working to 5 minute periods:
5 minutes to 7, etc.

Step 5. Preparation for more formal approach to time.

(*a*) *A day.* By questioning, gradually elicit that while we have space for 12 hours on our clock, there are 24 hours in a day, which is indicated by the fact that we go up to 12 twice, once in the morning and then after starting again we go up to 12 once more

in the afternoon and evening. If the children find any difficulty with this, it is easy to refer them to sun time, where in just the same way we go up to 12 once during the day and then up to 12 again during the night to get back to exactly the same time of day as that at which we started.

Thus we see that 24 hours make up day (and night).

(*b*) *a.m.* and *p.m.* Explain that these two terms are used to distinguish which lot of 12 hours we mean. If we are told, ' Come at 7 o'clock on Friday ', we may sometimes not be sure whether it is 7.0 a.m. (i.e. 7 in the morning) or 7.0 p.m. (i.e. 7 in the evening).

Explain that a.m. and p.m. are short for ante-meridian and post-meridian, which stand for, respectively, ' before the sun gets to its highest point ' and ' after the sun has left its highest point in the sky '.

(*c*) Presentation and learning of the formal table :

60 minutes = 1 hour
and 24 hours = 1 day

Step 6. Simple calculations in time.

(*a*) (i) First of all in hours. E.g. from 4 o'clock to 8 o'clock we can easily count round 1 (5), 2 (6), 3 (7), 4 (8) to discover that it is 4 hours, but from this and two or three similar examples it can soon be seen that it is quicker to subtract :

$$10 - 6 = 4 \text{ (hours)}$$

(ii) Now, while 11.0 a.m. is obviously 7 hours later than 4.0 a.m., the following presents a slight difficulty :

10 a.m. to 3.0 p.m.

By the counting method count round with the children. How many? (5)

Yet 3 – 10 does not look like 5

(and 10 – 3 = 7, so this is obviously the wrong method).

How many hours on the clock? (12)

So in this case we have to carry a complete unit from the ' clock hours ', just as we learnt to carry tens and hundreds right back in Class 2.

Now, however, the number of hours which must be carried is 12.

Thus we get :

$$\begin{array}{rlr} & \text{10 a.m. to 3.0 p.m.} & \\ +12 & 3.0 = \text{really} & 15.0 \\ & \underline{-10.0} & \underline{10.0} \\ & & 5.0 \text{ hours} \end{array}$$

(iii) Children do 20 simple quick calculations in hours across 12 o'clock as demonstrated in (ii) above.

(*b*) Calculations in minutes based on a 60 minute hour.

(i) Here it is necessary to ' unlearn ' (or modify) to some degree what we have already learnt, for we require for arithmetical calculation something rather different from what we use in ordinary English speech in telling the time.

In the latter we really use two separate lots of 30 minutes in each hour, those *past* and those *to* ; but this is not convenient in calculating, so we use the ' 60 minute hour ' where all minutes are *past* the hour until the next actual hour is reached. This may also be called ' timetable time ' (not just because it is used in school timetables, but in timetables of all types).

(ii) Children to be shown the large double clock again and compare (by counting), say :

10 past 6 = 6.10

and 10 to 7 = 6.50

5 past 6 = 6.05 *Note :* Note zero and explain.

and so on.

(iii) After several examples like this, and when the children obviously appreciate what is involved, the teacher sets 20 or more conversions as follows: quarter past five, 5 to 7, half past nine, 10 past 4, 25 to 12, 22 minutes to 12, etc.

(iv) The converse of this, changing what we might call ' calculation ' time into ordinary speech. Again 20 examples to be set by the teacher as follows :

1.20 ; 3.05 ; 9.45 ; 7.27 ; 12.45 ; 5.30, etc.

(*c*) Simple calculations in hours and minutes.

(i) Children do:

3.40	2.15	10.55	9.23	11.55
−1.20	−1.05	− 7.25	−6·16	−10.08

(ii) Now revise:

(*a*) How many minutes in an hour? (60)

(*b*) How many minutes on the clock face? (60)

(*c*) How many hours on the clock face? (12)

(iii) Explain carrying principle as being on the same basis as for hours above but now, obviously, we borrow 60 for minutes, and not 12 (as for hours). Also as in pure number, we must make allowance for these extra 60 minutes in the hours column.

Demonstrate this with two examples as follows:

(*a*) How long from 7.30 a.m. to 10.05 a.m.?

$$\begin{array}{r} 9\ \ 65 \\ \cancel{10.05} \\ -7.30 \\ \hline 2.35 \end{array} = 2 \text{ hr. } 35 \text{ min. } \textit{Ans.}$$

(*b*) How long from 9.15 a.m. to 4.45 p.m.?

$$\begin{array}{r} 12+\quad\ \ \\ 4.45 \\ -9.15 \\ \hline 7.30 \end{array} = 7 \text{ hr. } 30 \text{ min. } \textit{Ans.}$$

Include carrying figures in the demonstrations for clarity, but if children get them right straight away, let them do their own work without writing these down, as they should be able to carry mentally. If anyone has difficulty, however, get him to include these aiding figures for a while.

(iv) Children to do 10 of (iii) (*a*) above followed by 10 of (iii) (*b*), then 20 more mixed examples. All 40 to be set by the teacher.

Step 7. The four rules in time.

(*a*) Explain first of all that we probably shall not want, say, to add or divide times very often, but it is useful to know how to do so. Thus:

(*b*) (i) Teacher explains that:

We may wish to add time, if, say,

John walks home in $1\frac{1}{2}$ hours and Peter walks home in 1 hour and 5 minutes, and we wish to know how long they both take:

$$\begin{array}{rr} \text{hr.} & \text{min.} \\ 1 & 30 \\ +1 & 05 \\ \hline 2 & 35 \\ \hline \end{array} \quad \textit{Ans.}$$

or again

$$\begin{array}{rr} \text{hr.} & \text{min.} \\ 3 & 45 \\ +2 & 38 \\ \hline 6 & 23 \\ \hline 1 & 60\overline{)83} \\ & 1 \text{ r } 23 \end{array}$$

(*Note:* Under-the-line work for demonstration stage only, and to be dropped as soon as convenient.)

(ii) Children work 10 examples on these lines set by the teacher.

(iii) Increase to 3 and 4 items, teacher setting several examples of each for the children to do.

The teacher should note that as we are now merely adding in time and are not confined to a.m. and p.m. our answer may exceed 12 hours. It is recommended, however, that no answers exceed 24 hours (i.e. 1 complete day).

(*c*) *Multiplication.* (i) Teacher explains that if, say, 7 people each take half an hour to do something, we may wish to know how long they spend jointly on the job. Again we proceed here

exactly the same as in money, length, etc., remembering only our own table for time, which is:

60 minutes = 1 hour
and 24 hours = 1 day

Thus we get

hr.	min.
0	30
	$\times 7$
3	30

$$3 \qquad 6\cancel{0})21\cancel{0}$$
$$3 \text{ r } 30$$

or again

hr.	min.
1	43
	$\times 4$
6	52

$$\begin{array}{ll} 2 & \\ 4 & 6\cancel{0})172 \\ \hline 6 & \quad 2 \text{ r } 52 \end{array}$$

(ii) Teacher to set 10 examples for the children to do on these lines.

Rules: Short multiplication only (i.e. up to 12) and answers not to exceed 24 hours.

(*d*) Division.

(i) The teacher explains that if we have, say, $5\frac{1}{4}$ hours for lessons and we want to have nine lessons of equal length during the day, we could work out the length of each lesson as follows:

$$\begin{array}{ll} \text{hr.} & \text{min.} \\ & 315 \\ 9)5 & \cancel{15} \\ \hline & 35 \; \textit{Ans.} \end{array}$$

$$\begin{array}{r} 5 \\ \times 60 \\ \hline 300 \text{ min.} \end{array}$$

	hr.	min.
		168
or again	7)9	~~48~~
	1	24 *Ans.*

(ii) Teacher to set 20 simple examples for the children on these lines. *Rules:* Short division only and dividend not to exceed 24 hours.

Step 8. The calendar.

This step should preferably be dealt with concurrently in English lessons while Step 7 is proceeding in arithmetic lessons.

(*a*) Revise vocabulary already known, including, e.g. dial, hour hand, minute hand, o'clock, a.m., p.m., quarter past.

(*b*) Introduction and explanation of new terms :

i.e. (i) We know 60 minutes = 1 hour, and 24 hours = 1 day but now 7 days = 1 week.
Then learn names of all the days.

Continue (ii) 365 days = 1 year, but also 12 months = 1 year.
Then let children become familiar with names of the twelve months and their varying lengths in days.

(iii) Attempt to explain leap year.

(iv) Introduce the rhyme :

30 days hath September,
April, June and November,
All the rest have 31, excepting February alone
which has but 28 days clear
And 29 in each leap year.

Step 9. Introduction of seconds.

Apparatus. If possible, the teacher should obtain a watch with a second hand for this lesson.

(*a*) With such a watch it is easy to demonstrate (in groups if necessary) that whereas the minute hand makes a complete revolution for every hour, so the second hand makes a complete revolution for every minute.

The teacher explains that as there are 60 minutes in an hour so there are 60 seconds in a minute.

(*b*) It should be possible now to proceed fairly quickly in theory to simple examples in minutes and seconds:

min.	sec.	
5	15	
+20	55	
26	10	*Ans.*
1	60)70	
	1 r 10	

Children do 5 simple examples on each of addition, subtraction, multiplication and division of minutes and seconds, as set by the teacher.

(*c*) Proceed to work in 3 quantities, i.e. hours, minutes and seconds. Work through all 4 rules in turn, remembering the following *rules*: (i) maximum of 4 items in addition, (ii) short multiplication and division only.

On this basis the teacher should make up and set 10 examples for each rule.

Step 10. Introduction of the 'traveller's' or full timetable time, i.e. as used on railways, ships, aeroplanes, etc., and by armies, etc., and any other people who are likely to be active at any time of the 24 hours.

(*a*) Refer back to Step 6(*b*) above, where we say that 'timetable time' was based on a full 60 minute hour rather than the normal conversational way of expressing the time.

(*b*) From (*a*) above, it is easy to proceed to explain that in the same way timetable time works on a full 24-hour day and does not bother with a.m. and p.m., merely going straight on from the former to the latter by proceeding from 12.00 to 13.00 and continuing right through to 23.59 hours.

(*c*) Practice converting conversational and a.m. and p.m. times to timetable time, e.g.

10 to 5	(morning) =	4.50 a.m. =	4.50 hr.
10 to 5	(evening) =	4.50 p.m. =	16.50 „
half past 10	(morning) =	10.30 a.m. =	10.30 „
quarter to 6	(evening) =	5.45 p.m. =	17.45 „
half past 12	(night) =	12.30 a.m. =	0.30 „

and so on.

The teacher setting 20 or more examples for the children to convert.

(*d*) From this and a few simple examples it is soon clear that the only real difference that comes into our working now is that when it is necessary to ‘ borrow ’ any hours, we must take 24 now, rather than 12, as we did with a.m. and p.m.

(*e*) Children to work 10 examples for each of the 4 rules in timetable time in 3 quantities. These to be set by the teacher on the same general outlines as detailed above.

MONEY

1. £ AND SH.

Revision of Class 5 work. Increase gradually the difficulty of the numbers.

2. PROFIT AND LOSS.

Step 1. Revision of Class 5 work.

Revise the work in Class 5 :

(1) Profit and loss with internal multiplication

(2) C.P. = S.P. − Profit

(3) C.P. = S.P. + Loss

(4) S.P. = C.P. + Profit

(5) S.P. = C.P. − Loss

by mental work and questioning, e.g.

(*a*) I paid sh. 7 for 2 baskets and sold them for sh. 4.50 each. What was my profit?—sh. 2.

(*b*) I paid sh. 5 for a pot and made a loss of 50 ct. For how much did I sell it?—sh. 4.50.

(*c*) Musa made a profit of sh. 1.50 by selling a chair for sh. 9, What did he pay for it?

After each question ask how the class found the answer, thereby eliciting the rules.

Step 2. Now give written work on this section, giving problems increasing in difficulty.

PERCENTAGE

Step 1. The meaning of the word percentage.

Write $\frac{1}{100}$ on the blackboard. Ask the class what the fraction is. Answer : 'One over a hundred' or 'one hundredth'. Explain that this can be called 1 per cent. 'Cent is a Latin word

meaning hundred. (Hence cents in money—a hundred cents in one shilling.) Per cent means out of a hundred. So one per cent means one out of a hundred or one over a hundred or one hundredth, but we do not write one per cent as the fraction $\frac{1}{100}$, but as 1%. The sign % means per cent. If you look at the sign you will see that it is made up of the figures 100. $\frac{1}{100}$ is 1 of a hundred equal parts. This can be written as 1%.'

Step 2. Changing fractions with denominator 100 into percentage figures.

Write on the blackboard various fractions with denominator of a hundred and ask the percentage, e.g. $\frac{22}{100}$, $\frac{13}{100}$, $\frac{79}{100}$, etc., so that the blackboard reads:

$$\frac{22}{100}$$

$$\frac{13}{100}$$

$$\frac{79}{100}$$

As the class changes the fractions into percentage figures, the teacher writes the percentages opposite the fractions, e.g. $\frac{22}{100}=22\%$.

Step 3. Changing percentages to fractions.

Write 5% on the blackboard. Ask the fraction. Answer: '$\frac{5}{100}$'. Explain: 'This percentage can *only* mean 5 out of 100, it cannot mean 5 out of 200, etc., e.g. "40% of the villagers are children", means that 40 out of every 100 villagers are children, so if there are 300 villagers, 40 out of each 100 are children, that is 120 children (40 × 3). E.g. if Mary gained 70 marks out of 150 for her test, this would not mean that she gained 70%, because the total marks were 150, not 100.'

Step 4. Practical work on changing percentage into fractions.

Write various percentages on the blackboard and ask the class

to give fractions, e.g. 10%, 45%, 72%, 81%, etc. As the fractions are given, write on the blackboard thus:

$$10\% = \frac{10}{100}$$

$$45\% = \frac{45}{100} \text{ etc.}$$

Then with the class bring the fractions to their lowest terms, e.g. $10\% = \frac{10}{100}$. Question: 'Can we make the figures of this fraction smaller?' Answer: 'Yes, by cancelling by 10.' Do likewise for all the others. Work the cancellations on the blackboard and its resulting answer, so that the blackboard will read:

$$10\% = \frac{\overset{1}{\cancel{10}}}{\underset{10}{\cancel{100}}} = \frac{1}{10}$$

$$45\% = \frac{\overset{9}{\cancel{45}}}{\underset{20}{\cancel{100}}} = \frac{9}{20}$$

Step 5. Practical work on changing percentages to fractions.

Give at least 20 examples for written work: e.g. change the following percentages to fractions in their lowest terms:

15%; 50%; 25%; 80%; 44%; 100%

Step 6. Commonly known percentages and their equivalent fractions.

With the class work out the commonly known percentages into fractions, the teacher writing on the blackboard, and let them learn by heart. The blackboard should read as follows:

$$100\% = \frac{\cancel{100}}{\cancel{100}} = 1 \qquad 75\% = \frac{\overset{3}{\cancel{75}}}{\underset{4}{\cancel{100}}} = \frac{3}{4}$$

$$50\% = \frac{\overset{1}{\cancel{50}}}{\underset{2}{\cancel{100}}} = \frac{1}{2} \qquad 25\% = \frac{\overset{1}{\cancel{25}}}{\underset{4}{\cancel{100}}} = \frac{1}{4}$$

$$12\tfrac{1}{2}\% = \frac{12\frac{1}{2}}{100} = 12\tfrac{1}{2} \div 100 = 12\tfrac{1}{2} \times \frac{1}{100} = \frac{\cancel{25}}{2} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{1}{8}$$

A chart should already be prepared with the above percentages and their equivalent fractions written thus :

$$100\% = 1$$
$$75\% = \tfrac{3}{4}$$
$$50\% = \tfrac{1}{2}$$
$$25\% = \tfrac{1}{4}$$
$$12\tfrac{1}{2}\% = \tfrac{1}{8}$$

When the fractions above have been worked with the class, the chart should be hung on the wall. Then proceed with the next group of percentages in the same way. Blackboard reads :

$$20\% = \frac{\overset{1}{\cancel{20}}}{\underset{5}{\cancel{100}}} = \frac{1}{5} \qquad 10\% = \frac{\overset{1}{\cancel{10}}}{\underset{10}{\cancel{100}}} = \frac{1}{10}$$

$$5\% = \frac{5}{100} = \frac{1}{20}$$

$$2\tfrac{1}{2}\% = \frac{2\frac{1}{2}}{100} = 2\tfrac{1}{2} \div 100 = 2\tfrac{1}{2} \times \frac{1}{100} = \frac{\cancel{5}}{2} \times \frac{1}{\underset{20}{\cancel{100}}} = \frac{1}{40}$$

A second chart, as follows, should then be hung on the wall ·

$$20\% = \tfrac{1}{5}$$
$$10\% = \tfrac{1}{10}$$
$$5\% = \tfrac{1}{20}$$
$$2\tfrac{1}{2}\% = \tfrac{1}{40}$$

The third group of fractions is dealt with similarly. The blackboard reads :

$$66\tfrac{2}{3}\% = \frac{66\frac{2}{3}}{100} = 66\tfrac{2}{3} \div 100 = 66\tfrac{2}{3} \times \frac{1}{100} = \frac{\overset{2}{\cancel{200}}}{3} \times \frac{1}{\underset{1}{\cancel{100}}} = \frac{2}{3}$$

$$33\tfrac{1}{3}\% = \frac{33\frac{1}{3}}{100} = 33\tfrac{1}{3} \div 100 = 33\tfrac{1}{3} \times \frac{1}{100} = \frac{\cancel{100}}{3} \times \frac{1}{\cancel{100}} = \frac{1}{3}$$

The third chart should then be hung on the wall. This should read :

$$66\tfrac{2}{3}\% = \tfrac{2}{3}$$
$$33\tfrac{1}{3}\% = \tfrac{1}{3}$$

These three charts should remain on the wall throughout all the lessons dealing with percentage and should be memorised as early as possible.

Step 7. Finding percentages.

This is treated similarly to finding a fraction of a number, the percentage being changed into a fraction. Write the following sum on the blackboard :

Find 70% of 50

Question : 'What do we mean by 70%?'
Answer : '$\frac{70}{100}$.'
Question : 'Therefore, what does "find 70% of 50" mean?'
Answer : 'Find 70 hundredths of 50.'
Question : 'How will we do it?'
Answer : 'By multiplying $\frac{70}{100}$ by $\frac{50}{1}$.'

With the class work out on the blackboard :

$$\frac{70}{100} \times \frac{50}{1} = 35$$

Question : 'Therefore, when finding a percentage of a number, what must we do first?'
Answer : 'Change the percentage into a fraction.'
Question : 'What is the next step?'
Answer : 'Multiply the fraction by the number given.'

Work several examples on the blackboard with the class, making sure that the children follow the rules :

1. Change the percentage into a fraction.
2. Multiply the fraction by the number given.

Step 8. Practical work on Step 7.

Give at least 20 examples on Step 7, for written work :

(*a*) Table work first, e.g. Find (i) 25% of 150
(ii) $12\tfrac{1}{2}\%$ of 64
(iii) 75% of 428

Give at least 10 examples of this type.

(*b*) Other percentages, e.g. Find (i) 40% of 105
(ii) 55% of 40
(iii) 24% of 775

Give at least 10 examples of this type. Then give at least 10 examples where the answer contains a fraction as:

$$\text{Find } 35\% \text{ of } 630. \quad \frac{\overset{7}{\cancel{35}}}{\underset{2}{\cancel{100}}}\times\frac{63\cancel{0}}{1}=\frac{441}{2}=220\tfrac{1}{2}$$

Make certain that the children are sure of the mechanical working.

Step 9. Problem application.

Write the following problem on the blackboard:

> Mary gained a mark of 72% for her English test. The total marks for the paper were 150. How many marks did she gain?

Question : 'What have we to find?'
Answer : 'The number of marks gained by Mary.'
Question : 'What was the highest number of marks she could have gained?'
Answer : '150.'

Write on the blackboard:

Total marks = 150

Question : 'Did she gain full marks?'
Answer : 'No.'
Question : 'How many did she gain?'
Answer : '72% of the full marks.'

Write this on the blackboard so that the blackboard reads:

Total marks = 150
Percentage gained = 72%

Question : 'How will we find how many marks she gained?'
Answer : 'Multiply 72% by 150.'
Question : 'How will we do it?'
Answer : 'By changing 72% into $\frac{72}{100}$, then multiplying $\frac{72}{100}$ by 150.'

Work out on the blackboard with the help of the class, so that the blackboard reads :

$$\text{Total marks} = 150$$

$$\text{Percentage gained} = 72\%$$

$$\therefore \text{ Marks gained} = \frac{\overset{36}{\cancel{72}}}{\underset{2}{\cancel{100}}} \times \frac{\overset{3}{\cancel{150}}}{1} = 108 \text{ marks}$$

In this step, neat setting out of the problems should be stressed as the class are familiar with the mechanical process. Give at least 10 problems of this nature for written work, e.g. ' I have 280 banana trees. There is fruit on 30% of these. How many have fruit? '

Step 10. Changing fractions into percentages.

To change a fraction into a percentage means to find the number per cent which is equal to that fraction of 100, e.g. to change $\frac{3}{4}$ into a percentage, we find $\frac{3}{4}$ of 100%. The number found will be the percentage. To find a fraction of a number we must multiply the number by the fraction. Similarly, to find a fraction of 100%, we must multiply 100% by the fraction given. ' Per cent ' means out of 100, so the multiplicand in finding percentages is always 100%. The per cent sign (%) is treated in the same way as yd., gal., etc., giving its name to the answer. Therefore, to change $\frac{3}{4}$ into a percentage, we proceed as follows :

$$\frac{3}{\cancel{4}} \times \frac{\overset{25}{\cancel{100}}}{1}\% = 75\% \quad \textit{Ans.}$$

Work this on the blackboard with the help of the class. E.g. Change $\frac{13}{30}$ into a percentage. Work on blackboard with class :

$$\frac{13}{3\cancel{0}} \times \frac{10\cancel{0}}{1}\% = \frac{130}{3}\% = 43\tfrac{1}{3}\% \quad \textit{Ans.}$$

Step 11. Practical work on changing fractions into percentages.

Give at least 20 examples for written work on this step. E.g. change the following fractions into percentages :

(*a*) $\frac{20}{50}$	(*b*) $\frac{17}{25}$	(*c*) $\frac{51}{100}$	(*d*) $\frac{108}{180}$
(*e*) $\frac{288}{450}$	(*f*) $\frac{7}{8}$	(*g*) $\frac{7}{24}$	(*h*) $\frac{48}{72}$

Step 12. Problems using the mechanical working of Step 11. Write the following problem on the blackboard :

60 girls sat for an examination. Of these 48 passed. What percentage passed?

Question : 'What have we to find?'
Answer : 'The percentage of girls who passed the examination.'
Question : 'How many girls sat for the examination?'
Answer : '60.'

Write on the blackboard to read :

Number sat for examination = 60

Question : 'Did they all pass?'
Answer : 'No, only 48.'

Write on the blackboard so that blackboard reads :

Number sat for examination = 60
Number passed examination = 48

Question : 'If 48 out of 60 passed, how will we write the fraction?'
Answer : '$\frac{48}{60}$.'
Question : 'How will we find what percentage $\frac{48}{60}$ is?'
Answer : 'By multiplying $\frac{48}{60}$ by $\frac{100}{1}$%.'

Complete on the blackboard, class working, until the blackboard reads :

Number sat for examination = 60
Number passed examination = 48

$$\therefore \text{ Percentage passed } = \frac{\overset{16}{\cancel{48}}}{\underset{\cancel{3}}{\cancel{60}}} \times \frac{\overset{5}{\cancel{100}}}{1}\%$$

$= 80\%$ passed *Ans.*

When this has been understood, write the following problem on the blackboard :

Paul harvested 250 sacks of coffee from his shamba, and sold 245 of them. What percentage did he (*a*) sell, (*b*) keep?

Question : 'What have we to find?'
Answer : 'The percentage of sacks sold and kept.'

Say: 'We will deal with (*a*) first, that is, what percentage he sold.'

Question: 'How many sacks did Paul harvest?'
Answer: '250.'

Write statement and fact on the blackboard thus:

Number of sacks harvested = 250

Question: 'Did he sell them all?'
Answer: 'No, only 245 of them.'

Write on the blackboard the second statement and fact so that the blackboard reads:

Number of sacks harvested = 250
Number of sacks sold = 245

Question: 'Therefore, what fraction of the whole did he sell?'
Answer: '$\frac{245}{250}$.'
Question: 'How will we find the percentage of sacks he sold?'
Answer: 'Multiply $\frac{245}{250}$ by $\frac{100}{1}$%.'

With class, work this out on the blackboard, until blackboard reads:

Number of sacks harvested = 250
Number of sacks sold = 245

(*a*) ∴ Percentage sold $= \frac{\overset{49}{\cancel{245}}}{\underset{\cancel{5}}{\cancel{250}}} \times \frac{\overset{2}{\cancel{100}}}{1}\%$

$= 98\%$ sold

Say: 'We have found the answer to (*a*). Write (*a*) before the third statement. Now we must find the answer to (*b*), that is, what percentage did he keep.'

Write (*b*) on next line on the left.

Question: 'What does 250 sacks represent?'
Answer: '100%.'
Question: 'How much of this did he sell?'
Answer: '98%.'
Question: 'Therefore, what percentage did he keep?'
Answer: '2%.'
Question: 'How did you find the answer, 2%?'
Answer: 'By subtracting 98% from 100%.'

Work on the blackboard with the class, until blackboard reads :

$$
\begin{aligned}
&\text{Number of sacks harvested} &&= 250\\
&\text{Number of sacks sold} &&= 245\\
(a)\quad \therefore\ &\text{Percentage sold} &&= \frac{245}{250}\times\frac{100}{1}\%\\
& &&= 98\%\ \text{sold}\quad \textit{Ans.}\\
(b)\quad \therefore\ &\text{Percentage kept} &&= 100\% - 98\%\\
& &&= 2\%\ \text{kept}\quad \textit{Ans.}
\end{aligned}
$$

Give not less than 10 problems of these types.

PERCENTAGE PROFIT AND LOSS

This section should follow the work on Percentage.

Step 1. Percentage profit developed from previous knowledge of profit.

Ask : 'A man bought a cow for sh. 100 and sold it for sh. 110. What was his profit?'

Answer : 'Sh. 10.'

Question : 'What fraction of the C.P. was his profit?'

Answer : '$\frac{10}{100}$.'

Question : 'What percentage of the C.P. was his profit?'

Answer : '10%.'

Explain : 'Profit is always related to the C.P. So, if a man paid sh. 70 for an article and gained sh. 5 on selling it, his profit would be sh. $\frac{5}{70}$ of his C.P.

Ask : 'A dealer paid sh. 250 for a bicycle and sold it for sh. 290. What was his profit?'

Answer : 'Sh. 40.'

Question : 'What fraction of the C.P. was his profit?'

Answer : '$\frac{40}{250}$.'

Question : 'What was his percentage profit?'

Answer : '$\frac{40}{250}\times\frac{100}{1}\%$.'

Write on blackboard and work out with class help :

$$\frac{\overset{4}{\cancel{40}}}{\underset{1}{\cancel{250}}}\times\frac{\cancel{100}}{1}\% = 16\%$$

Step 2. The establishment of the formula of percentage profit.

Using the sum worked on the blackboard :

Question : 'What does the 40 represent?'
Answer : 'The profit in shillings.'

Write on blackboard at the right of the sum :

Profit

Question : 'What does 250 represent?'
Answer : 'The C.P. in shillings.'

Write C.P. on blackboard under 'Profit' like this :

$$\frac{\text{Profit}}{\text{C.P.}}$$

Question : 'What is the 100% for?'
Answer : 'To change the fraction into a percentage.'

Write 100% on blackboard so that blackboard reads :

$$\frac{\text{Profit}}{\text{C.P.}} \times \frac{100}{1}\%$$

Question : 'What does this find?'
Answer : 'Percentage profit.'

Write this answer on blackboard thus :

$$\text{Percentage Profit} = \frac{\text{Profit}}{\text{C.P.}} \times \frac{100}{1}\%$$

Step 3. Give written work on this step asking only to find the percentage profit.

E.g. (1) Joseph bought a sack of sugar for sh. 180 and sold it for sh. 216. What was his percentage profit?

E.g. (2) Paulo bought 3 cows for sh. 95, sh. 86 and sh. 89, and sold them all for sh. 300. What was his percentage profit?

Step 4. Establishment of the formula of percentage loss.

Deal with this in a similar way to percentage profit, obtaining the formula :

$$\text{Percentage Loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100\%$$

Step 5. Give written work on Step 4, asking only percentage loss.

E.g. Mark bought a roll of cotton material for sh. 96 and sold it for sh. 84. What was his percentage loss?

Make at least 10 examples for the class to work.

Step 6. Finding selling price given percentage profit.

Ask : 'What is 10% of sh. 50?

Answer : 'Sh. 5.'

Question : 'How did you find the answer?'

Answer : 'By multiplying sh. 50 by $\frac{10}{100}$.'

Question : If a man made a 10% profit on sh. 50, what would his profit be?'

Answer : 'Sh. 5.'

Question : 'Therefore, if the C.P. of an article was sh. 50, and the profit 10%, what would the selling price be?'

Answer : 'Sh. 55.'

Question : 'How did you find the selling price?'

Answer : 'By adding together the C.P. and the profit.

Write the following on the blackboard :

A trader wanted to make a 20% profit on his sugar. If he paid 55 ct. per lb. for the sugar, how much per lb. would he charge?

Question : 'What have we to find?'

Answer : 'The selling price.'

Question : 'How do we find the S.P.?'

Answer : 'By adding the C.P. and the profit.'

Question : 'Do we know the C.P.?'

Answer : 'Yes, 55 ct.'

Write on blackboard :

C.P. = 55 ct.

Question : 'Do we know the profit?'

Answer : 'No.'

Question : 'Can we find it?'

Answer : 'Yes, by finding 20% of 55 ct.'

Question : 'How will we do it?'

Answer : 'By multiplying 55 ct. by $\frac{20}{100}$.'

Write on blackboard and work with class help, so that blackboard reads:

$$\text{C.P.} = 55 \text{ ct.}$$

$$\text{Profit} = \overset{11}{\cancel{55}} \text{ ct.} \times \frac{\overset{1}{\cancel{20}}}{\underset{5}{\cancel{100}}} = 11 \text{ ct.}$$

Question : ' Can we find the S.P. now? '

Answer : ' Yes. By adding the C.P. and the profit.'

Complete sum on blackboard so that blackboard reads:

$$\text{C.P.} = 55 \text{ ct.}$$

$$\text{Profit} = \overset{11}{\cancel{55}} \text{ ct.} \times \frac{\overset{1}{\cancel{20}}}{\underset{5}{\cancel{100}}} = 11 \text{ ct.}$$

$$\text{S.P.} = 55 + 11 \text{ ct.}$$
$$= 66 \text{ ct. } \textit{Ans.}$$

Step 7. Give written work on Step 6.

E.g. A trader wanted to make 25% profit on his goods. What would he charge for the following if he paid:

(*a*) Potatoes - - 16 ct. per lb.
(*b*) Soap - - - 1/60 per bar
(*c*) Groundnuts - 80 ct. per lb.
(*d*) Butter - - sh. 3 per lb.

Step 8. Finding selling price given percentage loss.

Note: Traders do *not* aim to make a loss but sometimes they do lose on certain goods.

Work in similar way to Step 6.

E.g. A trader made a loss of 20% on his potatoes. If he paid 25 ct. per lb., how much per lb. did he charge?

And achieve the following setting:

$$\text{C.P.} = 25 \text{ ct.}$$

$$\text{Loss} = \overset{5}{\cancel{25}} \times \frac{\overset{1}{\cancel{20}}}{\underset{5}{\cancel{100}}} \text{ ct.} = 5 \text{ ct.}$$

$$\text{S.P.} = 25 \text{ ct.} - 5 \text{ ct.}$$
$$= 20 \text{ ct. per lb.}$$

Step 9. Give written work on Step 8, asking only S.P. given percentage loss.

E.g., Joseph bought a bicycle for sh. 320. It got damaged, so he sold it for a 25% loss. How much did he charge?

Step 10. With internal multiplication in the C.P.

Write on the blackboard :

John bought 4 goats for sh. 54, each.
He sold them, making a 10% profit. How much did he get?

Question : 'What have we to find?'
Answer : 'The selling price.'
Question : 'How do we find the S.P.?'
Answer : 'By adding the C.P. and the profit.'
Question : 'Do we know the C.P.?'
Answer : 'No.'
Question : 'How will we find it?'
Answer : 'By adding the prices of the goats.'

Work this on blackboard with class help, so that blackboard reads :

$$\begin{aligned} \text{C.P.} &= \text{sh. } 54 \times 4 \\ &= \text{sh. } 216 \end{aligned}$$

Question : 'Do we know the profit?'
Answer : 'No.'
Question : 'How will we find it?'
Answer : 'By multiplying the C.P., sh. 216, by $\frac{10}{100}$.'

Work on the blackboard with class help, so that blackboard reads :

$$\text{C.P.} = \text{sh. } 54 \times 4 = \text{sh. } 216$$
$$\text{Profit} = \text{sh. } \frac{216}{1} \times \frac{10}{100} = \text{sh. } \frac{216}{10} = \text{sh. } 21\tfrac{6}{10} = \text{sh. } 21.60$$

Question : 'Can we now find the S.P.?'
Answer : 'Yes, by adding the C.P. and the profit.'

Work on the blackboard so that it reads :

$$\begin{aligned} \text{C.P.} &= \text{sh. } 216 \\ \text{Profit} &= \text{sh. } 216 \times \tfrac{10}{100} = \text{sh. } \tfrac{216}{10} = \text{sh. } 21\tfrac{6}{10} = \text{sh. } 21.60 \\ \text{S.P.} &= \text{sh. } 216 + \text{sh. } 21.60 \\ &= \text{sh. } 237.60 \quad \textit{Ans.} \end{aligned}$$

Step 11. Give written work on this step.

E.g. (1) Petro bought 250 lb. of rice at 70 ct. per lb. He made a 20% profit. How much did he receive for the whole of it?

E.g. (2) Odira bought a sack of 1 cwt. of meal for sh. 75. He wished to make at least a 25% profit. How much per lb. did he charge?

DISCOUNT

Introduction

Before attempting to teach Discount it is essential that the pupils understand and can work percentages correctly. Discount deals with money percentages. *To avoid confusion in the minds of the pupils sh. and ct. should be changed to ct. before cancelling and the children then change the resulting cents answer back to shillings and cents.*

Find 5% of sh. 25.30.

Wrong way: sh. $\frac{25.3\cancel{0}}{1} \times \frac{\cancel{5}}{\underset{20}{\cancel{100}}} = $ sh. $\frac{2.53}{2} = $ sh. 1.26

The dot separating the 25 shillings and 30 cents may confuse the class, so that after cancelling they write it as sh. $\frac{25.3}{2}$ instead of sh. $\frac{2.53}{2}$ and when dividing, say :

'2 into 2 goes once, put down 1 ; 2 into 5 goes 2, remainder 1, put down 2 ; 2 into 13 goes 6, remainder 1, put down 6 ; add 0 ; 2 into 10 goes 5, put down 5.'

Then put the separating dot between the 2 and the 6, getting an answer of sh. 12.65 instead of sh. 1.26 (the remaining half cent being ignored). Hence it is better to work thus :

Right way: $\frac{253\cancel{0}}{1} \times \frac{\cancel{5}}{\underset{20}{\cancel{100}}}$ ct. $= \frac{253}{2}$ ct. $= 126\frac{1}{2}$ ct. $=$ sh. 1.26

The answer sh. 1.26 has been worked to the nearest whole cent.

Step 1. Mental drill on percentages.

Give simple mental sums on percentages :

(*a*) What is 10% of 20?
(*b*) What is 25% of 32?
(*c*) What is 75% of sh. 1?
(*d*) What is 50% of sh. 3?
(*e*) What is $\frac{1}{2}$ as a percentage? etc.

Step 2. Money percentages.

Write on the blackboard the following sum :

Find 5% of sh. 4

This is worked in the same way as percentages of numbers, remembering to change fractions of sh. 1 into cents.

Question : 'What does 5% mean?'
Answer : '$\frac{5}{100}$.'
Question : 'Therefore, what does the sum mean?'
Answer : 'Find 5 hundredths of sh. 4.'
Question : 'What will we do to find the answer?'
Answer : 'Multiply $\frac{5}{100}$ by sh. 4.'

Work out the sum on the blackboard with the class :

$$\frac{\cancel{5}}{\cancel{100}} \times \frac{\cancel{4}}{1} = \frac{1}{5} \text{ sh.}$$
$$\cancel{20}\ 5$$

Question : 'What is $\frac{1}{5}$ of sh. 1?'
Answer : '20 ct.'

Complete the sum so that it reads :

$$\frac{\cancel{5}}{\cancel{100}} \times \frac{\cancel{4}}{1} = \frac{1}{5} \text{ sh.} = 20 \text{ ct.}$$
$$\cancel{20}\ 5$$

This method should be used when shillings only are used, that is, working the sum in shillings and then changing the fraction to cents. When the class have understood, write the following on the blackboard :

Find 20% of sh. 6.25

Question : 'What have we to find?'
Answer : '20% of sh. 6.25.'
Question : 'How will we find it?'
Answer : 'By multiplying $\frac{20}{100}$ by sh. 6.25.'

Write first statement on the blackboard thus :

$$\text{sh. } \frac{6.25}{1} \times \frac{20}{100}$$

Say : 'In this sum we have two quantities—shillings and

cents. It is always better to work with one quantity only. In this case either shillings or cents. It is better to use cents.'

Question : 'If we want to change shillings and cents to cents, how do we do it?'
Answer : 'By removing the dot and calling it cents.'
Question : 'How many cents in sh. 6.25?'
Answer : '625 cents.'
Say : 'So using the quantity cents we write . . .'

Write on the blackboard after the first statement, so that the blackboard reads :

$$\text{sh. } \frac{6.25}{1} \times \frac{20}{100} = \frac{625}{1} \times \frac{20}{100}$$

Question : 'What are these now?'
Answer : 'Cents.'

Write cents after $\frac{625}{1} \times \frac{20}{100}$ and proceed so that blackboard reads :

$$\text{sh. } \frac{6.25}{1} \times \frac{20}{100} = \frac{\overset{125}{\cancel{625}}}{1} \times \frac{\cancel{20}}{\underset{5}{\cancel{100}}} \text{ ct.} = 125 \text{ ct.}$$

Question : 'What will we do with the answer?'
Answer : 'Change it to shillings and cents.'
Question : 'How do we change cents into shillings and cents?'

When the answer has been given, complete the sum on the blackboard so that it reads :

$$\text{sh. } \frac{6.25}{1} \times \frac{20}{100} = \frac{\overset{125}{\cancel{625}}}{1} \times \frac{\cancel{20}}{\underset{5}{\cancel{100}}} \text{ ct.} = 125 \text{ ct.} = \text{sh. } 1.25$$

Give the rule :

'Before multiplying by a fraction, change shillings and cents to cents.'

Step 3. Practical work on Step 2.

Give at least 20 examples on this step, no answer containing part of a cent.

E.g. (*a*) Find 10% of sh. 8.40.

(*b*) Find 25% of sh. 87.48.

(*c*) Find 30% of sh. 56.20.

Step 4. The teaching of the meaning of the word 'Discount'.

Put this statement on the blackboard :

A man bought a shirt which was marked sh. 28, but the trader sold it to him for sh. 26.

Question : 'Did the man pay the full price?'
Answer : 'No.'
Question : 'How much less did he pay?'
Answer : 'Sh. 2.'

Explain here that it is quite common for a trader to sell an article at a lower price than is marked. This may be done because the buyer is going to pay the full price immediately (this is called 'paying cash'), and to encourage people to pay cash the shopkeeper will often lower the marked price. In this case the owner lowered the price by sh. 2. This is called giving a discount of sh. 2. (Write *Discount* on the blackboard.) Discount is also given when large amounts are ordered, because shopkeepers like to sell a lot so that their profit is greater. This discount is often given to customers who pay cash or buy large amounts. Traders prefer to give the same amount of discount according to the amount bought. To do this the traders allow a certain percentage discount on the bill, e.g. East African traders often allow 5% discount to those who pay cash. This means that those who pay cash have $\frac{1}{20}$th of their bill subtracted, no matter how much the bill is. So discount is the taking away of a percentage of a bill.

Step 5. Finding discount.

Write on the blackboard the following sum :

Find the discount on a bill for sh. 10.70, if 10% discount is allowed.

Question : 'What have we to find?'
Answer : 'The discount on the bill.'
Question : 'What discount is allowed in this sum?'
Answer : '10% discount.'

Question : ‘ How will we find the discount? ’
Answer : ‘ Find 10% of sh. 10.70.’
Question : ‘ How will we write it to work it out? ’

As a child answers, the teacher will write on the blackboard thus :

$$\text{Discount} = \frac{10}{100} \times \frac{10.70}{1} \text{ sh.}$$

Question : ‘ Now what must we do? ’
Answer : ‘ Change the shillings and cents to cents.’

Ask a child to do it and then with the class work the sum on the blackboard so that it reads :

$$\text{Discount} = \frac{10}{100} \times \frac{10.70}{1} \text{sh.} = \frac{\cancel{10}}{\cancel{100}} \times \frac{107\cancel{0}}{1} \text{ ct.} = 107 \text{ ct.}$$
$$= \text{sh. } 1.07 \quad \textit{Ans.}$$

When the class are able to do so, the first statement, i.e. $\frac{10}{100} \times \frac{10.70}{1}$ sh. can be omitted so that they mentally change the shillings and cents to cents immediately.

Step 6. Ignoring the fractions of cents.

Write the following sum on the blackboard :

Find the discount of 5% on a bill for sh. 14.35

Question : ‘ What have we to find? ’
Answer : ‘ The discount.’
Question : ‘ How will we find it? ’
Anwer : ‘ By multiplying $\frac{5}{100}$ by cents 1435.’

Work out on the blackboard, with class help, to read :

$$\text{Discount} = \frac{\cancel{5}}{\underset{\underset{4}{\cancel{20}}}{\cancel{100}}} \times \frac{\overset{287}{\cancel{1435}}}{1} \text{ct.} = \frac{287}{4} \text{ ct.} = 71\tfrac{3}{4} \text{ ct.}$$

Explain : ‘ In this case the remaining fraction of a cent is ignored, so that the discount is 71 ct. Any fraction of a cent, no matter how large, is ignored by the trader.’

Step 7. Practical work on finding discount.

Give at least 10 examples of the following type :

(*a*) Find the discount on sh. 7.84 if the discount allowed is 10%.

(*b*) Find the discount on sh. 19.23 if the discount allowed is 5%.

(*c*) Find the discount of $2\frac{1}{2}\%$ on sh. 42.60.

Step 8. Finding the price paid after discount has been worked out.

Write the following sum on the blackboard :

Peter's bill was sh. 27.50. The trader allowed 10% discount for cash. How much cash would Peter pay?

Question : ' Would Peter pay more or less than sh. 27.50? '
Answer : ' Less.'
Question : ' How will we find how much less? '
Answer : ' By finding the discount.'
Question : ' How will we find the discount? '

As a child answers, write on the blackboard until it reads :

$$\text{Discount} = \frac{275\cancel{0}}{1} \times \frac{\cancel{10}}{\cancel{100}} \text{ ct.} = 275 \text{ ct.} = \text{sh. } 2.75$$

If the children are capable of doing so, the statement :

$$= 275 \text{ ct.}$$

may be omitted, so that the answer is given immediately in shillings and cents. Continue :

Question : ' If Peter pays less than the full amount, how will we find how much he pays? '
Answer : ' Subtract the discount from the bill.'

Work out on the blackboard, with class help, thus :

Peter's bill was	sh. 27.50
Discount was	sh. 2.75
∴ Peter paid	sh. 24.75 *Ans.*

Ask : ' Is there any quicker way of finding 10%? '

Answer : ' Divide by 10.'

If children can do this mentally, encourage them to do so, if not allow the working as above. Some children may feel more

confident working it out on paper. In this case it must be done as above, showing all the working in the body of the sum.

Step 9. Practical work on finding the amount paid.

Give five problems of the following type:

1. Everybody's Store allows 5% discount for cash. Find the amount of cash paid by customers on these bills:
   (*a*) sh. 89.30 (*b*) sh. 121.64 (*c*) sh. 400.

2. Bush grocers pay $2\frac{1}{2}$% discount for cash. How much will customers pay in cash on the following bills:
   (*a*) sh. 243.72 (*b*) sh. 17.80 (*c*) sh. 174.64

Step 10. Finding the percentage discount.

Write the following on the blackboard:

John's bill was sh. 146.80. He paid sh. 139.46 in cash. What was the discount?

Question: 'Did John pay the full amount?'
Answer: 'No. He paid less.'
Question: 'How will we know how much less?'
Answer: 'By finding the discount.'
Question: 'How will we find the discount in this case?'
Answer: 'By subtracting the smaller amount from the larger.'

Ask a child to do it, working on the blackboard, thus:

Amount of bill	sh. 146.80
Amount of cash paid	sh. 139.46
∴ Amount of discount	sh. 7.34

Now change the question on the blackboard so that the last sentence reads:

What was the percentage discount allowed?

Question: 'What are we asked now?'
Answer: 'To find the percentage.'
Question: 'Which percentage?'
Answer: 'The percentage sh. 7.34 is of the bill.'
Question: 'How will we do it?'
Answer: 'By multiplying the fraction by $\frac{100}{1}$%.'
Question: 'What is the fraction?'

As a child answers, write and work out on the blackboard so that the blackboard reads :

(Question) ———

$$\begin{array}{rlr} & \text{Amount of bill} & =\text{sh. } 146.80 \\ & \text{Amount of cash paid} & =\text{sh. } \underline{139.46} \\ \therefore & \text{Amount of discount} & =\text{sh. } \quad 7.34 \end{array}$$

$$\therefore \text{ Percentage discount } = \frac{\cancel{734}}{\underset{2}{\cancel{14680}}} \times \frac{\overset{5}{\cancel{100}}}{1}\%$$

$$=5\% \quad \textit{Ans.}$$

Work other examples with the class until they are certain of the process and the statements.

Step 11. Practical work on finding percentage discount.

Give at least 20 examples of the following type :

Jane's bill was sh. 273.40. She paid sh. 246.06. What was the percentage discount?

(When making a sum of this type, it is easier first to decide the percentage you require, e.g. 10%. Then find the bill amount, e.g. sh. 273.40, and by multiplying find the discount :

$$\frac{\cancel{10}}{\cancel{100}} \times 2734\cancel{0} = \text{sh. } 27.34$$

Then take the discount from the bill :

$$\begin{array}{r} \text{sh. } 273.40 \\ \text{sh. } \underline{27.34} \\ 246.06 \end{array}$$

which gives the cash amount paid. Then word your sum as above.)

INTEREST

This is a further application of percentage and should follow the previous work.

Step 1. Meaning of interest.

Build up the explanation of interest on the blackboard as follows. Say : ' John sold his bicycle, which was old, for sh. 100.

He wished to save to build a new house and so put this money in the bank.' Write on the blackboard :

Money John put in bank sh. 100

'The bank uses money put in to lend to people for building, starting shops, etc., and for being allowed to use it they pay the owners so much money each year. The bank into which John put his money paid him 5% more each year.' Write on the blackboard under previous statement :

Percentage paid each year 5%

Question : 'What does this mean?'
Answer : 'That they paid him sh. 5 for every sh. 100 he had in the bank, for one year.'
Question : 'Since John had sh. 100 in the bank, how much did they pay him at the end of the year?'
Answer : 'Sh. 5.'

Write on the blackboard under the first two statements :

Money paid to John for one year sh. 5

Question : 'Since John had sh. 100 in the bank, and they gave him an extra sh. 5, how much did he then have in the bank?'
Answer : 'Sh. 105.'

It should be made clear at this point that the money he put into the bank remains his, and that he can take out the money whenever he wishes. Continue :

Question : 'What did we do to find how much John was paid at the end of the year?'
Answer : 'We multiplied sh. 100 by 5%.'
Question : 'How can we write 5%?'
Answer : '$\frac{5}{100}$.'

Write on the blackboard :

$$\text{Money paid} = \text{sh. } \frac{\cancel{100}}{1} \times \frac{5}{\cancel{100}} = \text{sh. } 5$$

Explain, using the information on the blackboard :

1. 'The amount put into the bank is called the *principal*.' Insert after first statement on the blackboard, thus :

Money John put in bank (*principal*) sh. 100

2. 'The percentage paid each year is called the *rate*.' Again insert thus:

   Percentage paid each year (*rate*) 5%

3. 'The money given at the end of the year is called *interest*.' Insert on blackboard thus:

   Money paid for one year (*interest*) sh. 5

Ask: 'What can we write instead of "Money paid"?'
Answer: 'Interest.'

Write on the blackboard:

Interest

Ask: 'What was the $\frac{100}{1}$?'
Answer: 'Money put in the bank.'
Ask: 'Therefore what can we write instead of "Money put in bank"?'
Answer: 'Principal.'

Write on the blackboard:

Interest = Principal

Ask: 'What was the 5?'
Answer: 'The percentage paid each year.'
Ask: 'What can we write instead?'
Answer: 'Rate.'

Write on the blackboard:

$$\text{Interest} = \text{Principal} \times \text{Rate}$$

The percentage 5% is written as a fraction with numerator 100, so we write:

$$\text{Interest} = \text{Principal} \times \frac{\text{Rate}}{100}$$

We also write the principal as a fraction to avoid confusion, so it can be written thus:

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate}}{100}$$

Leave on the blackboard. Give an example to be worked on the blackboard with the class, e.g. the principal is sh. 150, the rate 10%, what will the interest be for 1 year?

Question: 'How will we find the interest?'
Answer: 'Multiply the principal by the rate and divide by 100.'

Write and work out on the blackboard, questioning the class throughout:

$$\text{Interest} = \text{sh. } \frac{15\cancel{0} \times \cancel{10}}{\cancel{100}} = \text{sh. } 15$$

E.g. The principal is sh. 120, the rate 3%, what will the interest be for one year? Work, as above, with class until:

$$\text{Interest} = \text{sh. } \frac{21\cancel{0} \times 3}{10\cancel{0}} = \frac{63}{10} = \text{sh. } 6\tfrac{3}{10}$$

Then ask: 'What is $\frac{1}{10}$ of sh. 1?'

Answer: '10 ct.'

Question: 'Therefore, what is $\frac{3}{10}$?'

Answer: '30 cents.'

Write the answer:

$$= \text{sh. } 6.30 \textit{ Ans.}$$

Give other examples, working with the children until they understand.

Step 2. Give at least 20 examples, making them interest for one year only, grading the difficulty:

(*a*) The principal is sh. 200, the rate is 5%, what will the interest be for one year? (This does not have a remainder to be changed to cents. Make 10 examples.)

(*b*) The principal is sh. 270, the rate 2%, what will the interest be for one year? (This has a remainder to be changed into cents. Make 10 examples like this.)

Step 3

(*a*) Give mental work on the previous process, e.g.:

1. The principal is sh. 200, the rate 1%, what will the interest be for one year? (sh.2)
2. The principle is sh. 150, the rate 3%, what will the interest be for one year? (sh. 4.50)

(*b*) Then ask: 'The principal is sh. 100, the rate 5%, what will the interest be for two years?' The class should be able to give the answer. Ask others so that all the class understand the

process, asking the interest for varying numbers of years, e.g. 5, 3, etc.

(*c*) Then ask how they found the answer. They answer: 'By multiplying the principal by the rate over 100 and multiplying by the number of years.' Explain that the 'number of years' is called the *time* (write on the blackboard). Ask for the formula given in the previous lesson:

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate}}{100} \quad \text{(Write on the blackboard)}$$

Say: 'So if we have a number of years to collect interest, we multiply by time, so that the formula becomes':

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

Complete on the blackboard.

Say: 'Instead of using the full words we use only the initial letter of each word, so that we say:

$$I = \frac{P \times R \times T}{100} \quad \text{(Write on blackboard)}$$

This should be written on a chart ready for the lesson and hung on the wall, until the whole topic is completed. Give examples to be worked on the blackboard with the class:

1. The principal is sh. 60, the rate 2%, what will the interest be for 5 years?

Ask the formula, and work the sum, questioning the class throughout, until the blackboard reads as follows:

$$I = \text{sh. } \frac{6\cancel{0} \times \cancel{2} \times \cancel{5}}{\cancel{100}} = \text{sh. } 6 \ \textit{Ans.}$$

$$2$$

2. The principal is sh. 465, the rate 4%, what will the interest be for 3 years?

This example is worked as above, but has a fraction of a shilling in the answer to be changed to cents. The sum is worked on the blackboard to this point, viz.:

$$I = \text{sh. } \frac{\overset{93}{\cancel{465}} \times \cancel{4} \times 3}{\underset{25\ \ 5}{\cancel{100}}} = \text{sh. } \frac{279}{5} = \text{sh. } 55\tfrac{4}{5}$$

Question : 'What is one-fifth of one shilling?'
Answer : '20 cents.'
Question : 'Therefore, what is four-fifths?'
Answer : '80 cents.'

Write the answer on the blackboard :

=sh. 55.80 *Ans.*

Step 4. Introducing the finding of the amount left after a number of years.

(*a*) Mental work revising first steps :

(1) Principal is sh. 300, rate 4%, what will the interest be after three years? etc.

(*b*) Put on the blackboard :

Paul put sh. 50 into the bank, the rate of interest was 2%, he left the money in for 5 years, how much interest would he have?

Answer : 'Sh. 5.'
Question : 'He then drew out all his money, how much did he receive?'
Answer : 'Sh. 55.'
Question : 'How did you find the answer sh. 55?'
Answer : 'By adding the principal and the interest.'

Give more simple mental work on this stage, the answers to which deal only in shillings.

(*c*) Put the following sum on the blackboard :

Musa put sh. 250 into the bank, the rate of interest was 3%, he left it in for 4 years, then drew out all his money. How much did he receive?

Question : 'What have we to find first?'
Answer : 'The interest for 4 years.'
Question : 'Then what have we to find?'
Answer : 'The total amount.'
Question : 'How will we find it?'
Answer : 'By adding the principal and the interest.'
Question : 'In what will the answer be?'
Answer : 'In shillings.'

Work out with class on the blackboard using the following setting to show all the information:

Principal = sh. 250

Rate = 3%

Time = 4 years

$$\therefore \text{ Interest} = \text{sh. } \frac{\overset{10}{\cancel{250}} \times 3 \times \cancel{4}}{\underset{25}{\cancel{100}}} = \text{sh. } 30$$

$$\therefore \text{ Total} = P + I = \text{sh. } 250 + 30 = \text{sh. } 280 \quad \textit{Ans.}$$

Work out several examples on the blackboard, working with the class, so that the children learn the setting out. Include in your blackboard examples ones having cents in the interest, e.g.:

> Mary put sh. 116 in the bank and left it in for 8 years. She was paid interest at the rate of 5% per year. After this time she drew out the money. How much did she receive?

Question as above.

Setting:

Principal = sh. 116

Rate = 5%

Time = 8 years

$$\therefore \text{ Interest} = \text{sh. } \frac{116 \times \cancel{5} \times \overset{2}{\cancel{8}}}{\underset{\cancel{25}\ 5}{\cancel{100}}} = \text{sh. } \frac{232}{5} = \text{sh. } 46\tfrac{2}{5} = \text{sh. } 46.40$$

$$\therefore \text{ Total} = P + I = \text{sh. } 116 + \text{sh. } 46.40 = \text{sh. } 162.40 \quad \textit{Ans.}$$

DECIMALS

In Class 5, the pupils should have become thoroughly familiar with the mechanical handling of decimals in four rules up to 2 decimal places. In Class 6, it will be found very useful if their ability is extended to the handling of 4 places of decimals, and if they know the decimal equivalents of the principal vulgar fractions.

Step 1. Introduction of third and fourth decimal places.

Begin by blackboard revision of the meaning of the first two decimal places:

$$\frac{1}{10} = \cdot 1, \text{ the first decimal place}$$

$$\frac{1}{100} = \cdot 01, \text{ the second decimal place}$$

At this point, Class 6 should be able to tell you in answer to questions the matter of the remaining part of the blackboard summary:

$$\frac{1}{1000} = \cdot 001, \text{ the third decimal place}$$

$$\frac{1}{10{,}000} = \cdot 0001, \text{ the fourth decimal place}$$

You may point out that the 'number of 0's in the denominator is the same as the number of decimal places when the denominator is 10, 100, 1000 or 10,000'. Give plenty of oral practice in translating $\frac{9}{1000}$, $\frac{7}{10000}$, etc., into decimals. You will also find that it is useful to put a chart—shown below—on the class-room wall and give the class practice in writing down the decimal equivalents of vulgar fractions read out by yourself.

Denominator:	10	100	1000	10,000
Numerator:				
1	·1	·01	·001	·0001
2	·2	·02	·002	·0002
3	·3	·03	·003	·0003
4	·4	·04	·004	·0004
5	·5	·05	·005	·0005
6	·6	·06	·006	·0006
7	·7	·07	·007	·0007
8	·8	·08	·008	·0008
9	·9	·09	·009	·0009

Step 2. Practice in 3 and 4 places.

(*a*) **Addition.** There is little difficulty in addition provided stress is laid on the need for neat layout of sums. Give about 20 sums of three or four items where the number of decimal places is not the same in each item:

$$156{\cdot}321 + 27{\cdot}1036 + 259{\cdot}9 + 7{\cdot}24, \text{ etc.}$$

(*b*) **Subtraction.** Revise the Class 5 matter in '0 Difficulties'. Then give graded practice as follows—several examples of each for the class to do:

(i) 396·2524 − 258·2626 (four from four places).
(ii) 1538·375 − 976·4127 (four from three).
(iii) 972·46 − 249·3289 (four from two).
(iv) 542·9 − 265·1452 (four from one).
(v) 874· − 353·2563 (four from no place).

Note: There is a lot of carrying across 0 in these sums. Watch it.

(*c*) **Multiplication.** Begin by multiplying decimals only after revising the rule learnt in Class 5. Grade examples like this:

·24 × ·87	·632 × ·9	·7 × ·496
·42 × ·25	·325 × ·4	·8 × ·375 (final 0's)
* ·24 × ·37 (*a*)	·08 × ·12 (*b*)	·03 × ·02 (*c*)

* A new difficulty—that of having to write in a 0—or two or three 0's—immediately after the decimal point to produce the correct answer. When the class has had practice on the first two grades, demonstrate the new type by reference to multiplication of fractions:

$$\begin{array}{r} \cdot24 \times \cdot37 \\ 24 \\ 37 \\ \hline 168 \\ 720 \\ \hline 888 \\ \hline \end{array}$$

'According to rule we need 4 decimal places in the answer,

but have only 3 digits. If we multiply our decimals as fractions, we shall see what to do.'

$$\frac{24}{100}\times\frac{37}{100}$$

$$=\frac{888}{10{,}000}$$

$$=\frac{8\cancel{0}\cancel{0}}{10{,}0\cancel{0}\cancel{0}}+\frac{8\cancel{0}}{10{,}00\cancel{0}}+\frac{8}{10{,}000}$$

$$=\quad \cdot08+\quad\cdot008+\cdot0008$$

$$=\cdot0888$$

'So, when I have not enough digits to make up the correct number of decimal places, I add 0s between the decimal point and the first figure.'

Give at least five examples of type (*a*): then of the second type marked (*b*): then of the third type marked (*c*).

Step 2. Multiplication to 4 places (*continued*)

When this basic work has been done, give several sums to the class to do, grading them according to the following:

(i) $29\cdot42 \times 37\cdot56$	(ii) $329\cdot4 \times 2\cdot743$
(iii) $9\cdot748 \times 537\cdot2$	(iv) $2976 \times \cdot7431$
(v) $76\cdot03 \times \cdot06$	(vi) $453\cdot5 \times \cdot006$
(vii) $6549 \times \cdot0008$	

Include some of the 0 difficulties in your examples, and also various examples working to 3 places only in the answer.

At this stage, pupils should be permitted individually—as you think they are able to understand—to drop meaningless 0s from the final statement of the answer. E.g. an answer may appear as 476200, then 47·6200, and be written finally as 47·26.

Division. There is no need for a detailed statement here to guide you. Take the steps given in division for Class 5, and extend them in turn to 3 and then 4 decimal places. Make sure that your answers, as well as divisors and dividends, never go beyond 4 decimal places, and before you start revise the rules for working division sums given in Class 5.

Step 3. Fractional values.

This 'step' can be carried out in tables and mental time when other work is being done. The class should know the following facts:

$\frac{1}{10} = \cdot 1$ $\quad$ $\frac{1}{5} = \cdot 2$...and the values of $\frac{3}{10}$, $\frac{2}{5}$, etc.

$\frac{1}{2} = \cdot 5$

$\frac{1}{4} = \cdot 25$

$\frac{1}{8} = \cdot 125$...and should be able to work out the values of $\frac{3}{4}$, $\frac{3}{8}$, etc.

and that any fraction whose denominator is a multiple of 3 cannot be expressed properly as a decimal. They should then have practice in dividing whole numbers by 5, 4, 8, and expressing the remainder as a decimal. It is valuable to make them compose their own 'tables' by working them out, e.g.:

```
   ·375
8)3·0
  2 4
  ---
    60        3÷8 or 3/8=·375
    56
    --
     40
     40
     --
```

PROPORTION

To teach these proportion sums clearly and well it is necessary to start from the Unitary Method sum learnt in Class 5.

Step 1. The elimination of the second or unit statement.

Put the following sum on the blackboard:

6 goats cost sh. 192; what is the cost of 10 goats?

Divide the blackboard into 3 equal sections by drawing two lines down it. With the aid of the class this sum is then done in the left-hand section of the blackboard which reads as follows:

(1st column)	(2nd column)	(3rd column)
6 goats cost sh. 192		
1 goat costs sh. $\frac{192}{6}$		
10 goats cost sh. $\frac{192 \times 10}{6}$		

On the first line of the second section he repeats the first statement and then says to the class:

'Let us try to do this sum combining the second statement into the third statement. We could write 10 goats cost *the cost of* 1×10.'

Write this on the blackboard as the second statement in the second section and ask:

'What is the cost of 1 goat?' sh. $\frac{192}{6}$

'What is the cost of 10 goats?' $\frac{192}{6} \times$ sh. 10

Write this on the blackboard so that it now reads:

(1st column)	(2nd column)	(3rd column)
6 goats cost sh. 192	6 goats cost sh. 192	
1 goat costs sh. $\frac{192}{6}$	10 goats cost *the cost of* $1 \times$ sh. 10	
10 goats cost sh. $\frac{192 \times 10}{6}$	$= \frac{192}{6} \times$ sh. 10	

Now say to the class:

'Let us try to write out this shortened form in a less awkward way.'

He again writes the first statement, this time in the third section. He says to the class:

'Before writing the second or final statement, decide what fraction is the cost of 1; then write the final statement remembering that the answer part of the cost of 1 will always be *multiplied* by the new figure in the beginning of this final statement.'

Do this on the blackboard so that it reads:

(1st column)	(2nd column)	(3rd column)
6 goats cost sh. 192	6 goats cost sh. 192	6 goats cost sh. 192
1 goat costs sh. $\frac{192}{6}$	10 goats cost *the cost of* $1 \times$ sh. 10	10 goats cost sh. $\frac{192}{6} \times 10$
10 goats cost sh. $\frac{192}{6} \times 10$	$= \frac{192}{6} \times$ sh. 10	= sh. 320

Now say to the class:

'Would you expect 10 goats to cost more than 6 goats, if each goat cost the same?'

'More.'

'Is the answer to our sum more?'

'Yes.'

'Does it seem a reasonable answer?'

'Yes.'

Emphasise to the class that this examination of the answer is important ; that they must, before they start the sum, have made an estimation of the answer. Should they then have an answer which is wrong through an error of reasoning, it will be noticed.

Now put the following sum on the blackboard :

5 cows cost sh. 700 ; what is the cost of 3 cows?

Ask the class, first, is the answer going to be more or less than sh. 700 and get from them some estimates. Then proceed to do the sum on the blackboard with the children. The teacher should do little in this—the class should be found capable.

Now do a third example on the blackboard with the children, this time *not* a ' cost ' sum. E.g. If I use 2 lb. of sugar in 3 weeks, how much will I use at this rate in a year?

Give the class at least 20 examples, with money, weight, capacity and length all involved in one or the other.

The class could now be told that this kind of sum, in which the unit line is not used, is called a Proportion sum.

WORK SUMS—INVERSE PROPORTION

These should be taught in the same way as the proportion sums, by leading from the Unitary Method.

Put this sum on the blackboard :

3 men dig a hole in 15 days, how long will 5 men take?

Get an estimation of the answer from the children.

Now divide the blackboard into 3 sections, and after doing the sum in the Unitary Method in the first section, go on in the manner as explained in the lessons on proportion. At the end of this the blackboard should read as :

(1st column)	(2nd column)	(3rd column)
3 men dig a hole in 15 days	3 men dig a hole in 15 days	3 men dig a hole in 15 days
1 man digs a hole in 15 × 3 days	5 men dig a hole in *time of* 1 *man* ÷ 5 days	5 men dig a hole in $\frac{\overset{3}{\cancel{15}} \times 3}{\underset{1}{\cancel{5}}}$ days
5 men dig a hole in $\frac{15 \times 3}{5}$ days		= 9 days

Point out how, as we found in Class 5, work sums *divide* by the new figure of the last statement, whereas other proportion sums *multiply*.

Further, point out that in work sums, as the number of people increases, the answer decreases ; as the number of figures

decreases, the answer increases—or, when the figure on the left increases, the answer decreases, which as already pointed out is the opposite to the proportion sums done before. For this reason these sums are known as *inverse proportion*, inverse meaning opposite.

Do another of these on the blackboard.

Give at least 20 sums of this type.

Follow with revision-practice in which both Proportion and Inverse Proportion are mixed.

REVISION

The third term of Class 6 has been left free for revision of Arithmetic in preparation for (at present) the Primary Schools Leaving Examination.

Teachers should see first that there is a thorough revision of the basic mechanical processes used in Arithmetic. This can best be done by taking examples from the work done in Classes 4, 5 and 6, so as to build a scheme of revision work as follows :

(i) Number —Addition, Subtraction, Short and Long Multiplication.

(ii) Number —Short and Long Division.

(iii) Money —First three rules as Number.
Money —Division as Number.

(iv) Length —Addition and Subtraction.
Length —Multiplication.
Length —Division.
Length —Division of Length by Length.

(v) Perimeter.

(vi) Area.

(vii) Capacity —In the same order as Number.

(viii) Weight —In the same order as Number.

(ix) Fractions—Four rules in order.

(x) Decimals—Four rules in order.

This work should be followed by a series of revision periods on those topics, such as Discount and Percentage, *following* the order in which they have been introduced into the arithmetic scheme, and insisting on a thorough drill of any formulae and accurate knowledge of terms involved.

The teacher should make, before the beginning of this term, a

parallel scheme of revision for all arithmetical tables, beginning with the very simplest. These should be drilled in part every day.

The best method of revising is to set first a short series of sums on the topic to be revised, and then to base any necessary explanation on study and analysis of any errors which may have been made by the class.

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